Kaon condensation in the color-flavor-locked phase of quark matter, the Goldstone theorem, and the 2PI Hartree approximation

Jens O. Andersen and Lars E. Leganger*

Department of Physics, Norwegian Institute of Science and Technology, N-7491 Trondheim, Norway
(Dated: October 30, 2008)

At very high densities, QCD is in the color-flavor locked phase, which is a color-superconducting phase. The diquark condensates break chiral symmetry in the same way as it is broken in vacuum QCD and gives rise to an octet of pseudo-Goldstone bosons and a superfluid mode. The lightest of these are the charged and neutral kaons. For energies below the superconducting gap, the kaons are described by an $O(2) \times O(2)$-symmetric effective scalar field theory with chemical potentials. We use this effective theory to study Bose-condensation of kaons and their properties as functions of the temperature and the chemical potentials. We use the 2-particle irreducible effective action formalism in the Hartree approximation. The renormalization of the gap equations and the effective potential is studied in detail and we show that the counterterms are independent of temperature and chemical potentials. We determine the phase diagram and the medium-dependent quasiparticle masses. It is shown that the Goldstone theorem is satisfied to a very good approximation. The effects of imposing electric charge neutrality is examined as well.

PACS numbers: 11.15Bt, 04.25.Nx, 11.10Wx, 12.38Mh

I. INTRODUCTION

There has been a large effort in recent years to map out the phase diagram of QCD as a function of temperature and baryon chemical potential [1, 2, 3, 4, 5, 6, 7, 8]. For example, much work has been done at high baryon density and the understanding of this part of the phase diagram has improved significantly as compared to one or two decades ago. At sufficiently high density and low temperature, we know that QCD is in the color-flavor locked (CFL) phase [1, 2, 3, 7]. This state is a color superconducting state since the quarks form Cooper pairs as electrons in an ordinary superconductor. The attraction between the quarks, which renders the Fermi surface unstable against the formation of Cooper pairs, is provided by one-gluon exchange.

At asymptotically high densities, one can ignore the strange-quark mass and quarks of all three colors and all three flavors participate in a symmetric manner in the pairing. The original symmetry group $SU(3)_{\rm c} \times SU(3)_L \times SU(3)_R \times U(1)_B$ is broken down to $SU(3)_{c+L+R}$ which is a linear combination of the generators of the original group. This linear combination locks rotation in color space with rotations in flavor space and this has given the name to the phase. In the CFL phase there is an octet of Goldstone modes which arises from the breakdown of chiral symmetry and a singlet arising from the breakdown of the baryon-number group $U(1)_B$. The latter is a superfluid mode since it is responsible for the superfluidity in the CFL phase. This is analogous to the superfluidity encountered in Bose-Einstein condensed phases in condensed-matter systems. Since the symmetry-breaking pattern is the same as in vacuum QCD, the low-energy properties of the CFL phase can be described in terms of an effective chiral Lagrangian for the octet of (pseudo)-Goldstone modes and the superfluid mode [9, 10, 11, 12, 13]. An important difference between chiral perturbation theory in the vacuum and in the CFL phase is that the latter is at high density and the Lagrangian is therefore coupled to chemical potentials via the zeroth component of a "gauge field".

At asymptotically high densities, all nine modes are exactly massless since one can ignore the quark masses. At moderate densities, this is no longer the case. The quark masses can not be neglected and chiral symmetry is explicitly broken. This implies that only the superfluid mode is exactly massless, while the other mesonic modes acquire masses. This is relevant for the interior of a neutron star. In this case, the quark chemical potential is of the order of 500 MeV, while the strange quark mass is somewhere between the current quark mass of approximately 100 MeV and the constituent quark mass of approximately 500 MeV [14]. The mass spectrum in the CFL is the opposite of vacuum QCD and the lightest massive modes are expected to be the charged and neutral kaons K^+/K^- and $K^0/\bar{K}^0$. Furthermore if the chemical potential associated with one of the bosons is larger than its vacuum mass, it will Bose condense. Various aspects of condensation of kaons in the CFL phase have been studied within Nambu-Jona-Lasinio models [15, 16, 17, 18, 19, 20, 21, 22].

Some of the properties of the kaons in the CFL phase have been studied using effective scalar field theories.

*Electronic address: andersen@tf.phys.no; Electronic address: lars.leganger@ntnu.no

For example, it has been shown that the symmetry-breaking that accompanies Bose condensation of kaons gives rise to unconventional Goldstone bosons if the original symmetry group is $SU(2) \times U(1)$ [23, 24, 25]. This is the case when the chemical potential for the neutral kaons is the same as that of the charged kaons. The Goldstone mode has a quadratic dispersion relation for small values of the three-momentum instead of the usual linear dispersion relation. This unsual property of the Goldstone mode arises from the lack of Lorentz invariance due to finite density.

In contrast to hadronic matter in heavy-ion collisions, bulk matter in compact stars must (on average) be electrically neutral and so a neutrality constraint must be imposed [26, 27]. Similarly, bulk matter must be color neutral and if the system is in a color superconducting phase, one sometimes has to impose this constraint explicitly. It is automatically satisfied if one uses the QCD Lagrangian, but not so if one describes the system using NJL-type models. If there are dynamical gauge fields present, the zeroth components of A^a_μ develop nonzero expectation values, $\langle A^a_0 \rangle \neq 0$, that effectively act as chemical potentials. In the NJL models there are no gauge fields present and the $SU(N_c)$ color symmetry is global [22, 28, 29, 30]. One must therefore impose these constraints explicitly.

One of the most popular approaches to the study of systems at finite temperature and density is the 2-particle irreducible (2PI) effective action formalism developed by Cornwall, Jackiw and Tomboulis [31] in the context of relativistic field theories. See e.g. Refs. [32, 33, 34, 35, 36, 37, 38, 39]. for various applications. The 2PI functional depends on background fields ϕ_0 and the exact propagator D. In practical calculations one must truncate the exact generating 2PI functional according to some approximation scheme. The $1/N$-expansion and the loop expansion are two such systematic schemes. These approximation schemes are nonperturbative in the sense that they sum up loop diagrams from all orders in the loop expansion. The renormalization of such approximations is therefore a nontrivial issues as standard theorems from perturbation theory do not apply. There has been significant progress regarding the renormalization of these approximations starting with the papers by van Hees and Knoll [40]. They showed that the equation for the two-point function in scalar ϕ^4-theory can be renormalized by introducing a finite number of counterterms. A more systematic study of the renormalization issues in scalar ϕ^4-theory was presented in Ref. [41]. They formulated an iterative renormalization procedure that determines the counterterms needed to eliminate the (sub)divergences. These counterterms were shown to be independent of temperature. In Ref. [42] the authors have developed a "direct" renormalization procedure which is equivalent to the iterative procedure in [41]. Later it has been shown [43] how to fix all the counterterms needed to calculate the proper vertices which are encoded in the effective action. These counterterms are local and they are independent of temperature and chemical potential.

The 2PI effective action formalism for scalar fields has recently been used to study the thermodynamics of pions and kaons and their condensation. In Ref. [39], the quasi-particle masses and the phase diagram is studied to leading order in the $1/N$ expansion of $O(N)$-symmetric scalar field theories. In Ref. [14], the authors applied the 2PI effective action formalism in the Hartree approximation to an effective $O(2) \times O(2)$-symmetric scalar field and calculated the phase diagram and the critical temperature for Bose-condensation of kaons. The effects of imposing electric charge neutrality were also investigated. The scalar theory for the kaons were derived from the effective chiral Lagrangian, where the parameters depend on the baryon chemical potential. Renormalization issues were not addressed. In the present paper, we reconsider the problem of kaon condensation from a somewhat different angle, and consider in some detail the renormalization of the theory. We also show that the violation of Goldstone's theorem is negligible.

The paper is organized as follows. In Sec. II, we discuss the 2PI Hartree approximation for a $O(N)$-symmetric Bose gas. In Sec. III, we briefly discuss $O(2) \times O(2)$-symmetric models, which are relevant for kaon condensation in the CFL phase of dense quark matter. We determine the quasi-particle masses as well as the phase diagram. Finally, we study the effects of imposing electric charge neutrality. In Sec. IV we summarize and conclude. In appendix A, we discuss the renormaliztion of the gap equations and effective potential in detail.

II. $O(2N)$-SYMMETRIC BOSE GAS

The Euclidean Lagrangian for a Bose gas with N species of massive charged scalars is

$$\mathcal{L} \;=\; (\partial_\mu \Phi_i^\dagger)(\partial_\mu \Phi_i) + m^2 \Phi_i^\dagger \Phi_i + \frac{\lambda}{2N}\left(\Phi_i^\dagger \Phi_i\right)^2 \;, \quad (1)$$

where $i = 1, 2, ..., N$ and $\Phi_i = (\phi_{2i-1} + i\phi_{2i})/\sqrt{2}$ is a complex field. The theory described by Eq. (1) has $(2N-1)N$ conserved charges which equals the number of generators of the group $O(2N)$.

A gas of N species of bosons can be characterized by the expectation values of the different conserved charges in addition to the temperature. For each conserved charge Q_i, one may introduce a nonzero chemical potential μ_i. However, it is possible to specify the expectation values of different charges only if they commute. The maximum number of commuting charges is N [44]

and these can be chosen as

$$Q_i \;=\; \int d^3x\, j_i^0 \;, \tag{2}$$

where the the current densities j_i^μ are

$$j_i^\mu \;=\; \phi_{2i}\partial^\mu\phi_{2i-1} - \phi_{2i-1}\partial^\mu\phi_{2i} \tag{3}$$

The incorporation of a conserved charge Q_i is done by making the substitution

$$\partial_0\Phi_i \to (\partial_0 - \mu_i)\,\Phi_i \tag{4}$$

$$\partial_0\Phi_i^\dagger \to (\partial_0 + \mu_i)\,\Phi_i^\dagger \;. \tag{5}$$

in the Lagrangian (1). Note that the chemical potential acts as the zeroth component of a gauge field.

From the path-integral representation of the thermodynamic potential Ω

$$e^{-\beta V\Omega} \;=\; \int \mathcal{D}\Phi_i^*\mathcal{D}\Phi_i e^{-\int_0^\beta d\tau\int d^3x\mathcal{L}} \;, \tag{6}$$

the expression for the charge density can be written as

$$Q_i \;=\; -\frac{\partial\Omega}{\partial\mu_i}\,. \tag{7}$$

If we introduce k chemical potentials, the full symmetry group is broken down to $[O(2)]^k\times O(2N-2k)$. If $m^2<0$, the $O(2N)$ symmetry is spontaneously broken down to $O(2N-1)$. Even if $m^2>0$, the symmetry may be broken if one of the chemical potentials, μ_i, is larger than a critical chemical potential $\mu_c = m$. In that case, the $\langle 0|\phi_{2i-1}|0\rangle \neq 0$ and the corresponding $O(2)$ symmetry is broken. In the following, we consider the simplest example of a single chemical potential $\mu=\mu_i$ for the complex field Φ_1. We first introduce a nonzero vacuum expectation value ϕ_0 for the field Φ_1 in order to allow for a charged condensate. Using the $O(2)$-symmetry, we can always choose ϕ_0 real and so we can write

$$\Phi_1 \;=\; \frac{1}{\sqrt{2}}\left(\phi_0+\phi_1+i\phi_2\right) \;, \tag{8}$$

where ϕ_1 and ϕ_2 are quantum fluctuating fields.

The inverse tree-level propagator then reads

$$D_0^{-1}(\omega_n,p) = \begin{pmatrix} \omega_n^2+p^2+m_1^2-\mu^2 & -2\mu\omega_n & 0 & 0 & ... \\ 2\mu\omega_n & \omega_n^2+p^2+m_2^2-\mu^2 & 0 & 0 & ... \\ 0 & 0 & \omega_n^2+p^2+m_3^2 & 0 & ... \\ 0 & 0 & 0 & \omega_n^2+p^2+m_3^2 & ... \\ ... & ... & ... & ... & ... \end{pmatrix} , \tag{9}$$

where $\omega_n = 2\pi nT$ are the Matsubara frequencies and the tree-level masses are

$$m_1^2 \;=\; m^2+\frac{3\lambda}{2N}\phi_0^2 \;, \tag{10}$$

$$m_2^2 \;=\; m^2+\frac{\lambda}{2N}\phi_0^2 \;, \tag{11}$$

$$m_3^2 \;=\; m^2+\frac{\lambda}{2N}\phi_0^2 \;. \tag{12}$$

The tree-level dispersion relation is found by analytic continuation to Minkowski space, $\omega_n\to i\omega$, and then solving the equation $\mathrm{Det}D_0(\omega,p)=0$. This yields

$$\omega_{1,2}(p) \;=\; \sqrt{p^2+\frac{1}{2}(m_1^2+m_2^2)+\mu^2 \pm \sqrt{4\mu^2\left[p^2+\frac{1}{2}(m_1^2+m_2^2)\right]+\frac{1}{4}\left(m_1^2-m_2^2\right)^2}} \;, \tag{13}$$

$$\omega_3(p) \;=\; \sqrt{p^2+m_3^2}\,. \tag{14}$$

The classical effective potential is

$$V \;=\; \frac{1}{2}\left(m^2-\mu^2\right)\phi_0^2+\frac{\lambda}{8N}\phi_0^4\,. \tag{15}$$

The minimum of the classical potential V is given by

$(m^2-\mu^2)+\lambda/2N\phi_0^2=0$ and at the minimum, we have $m_1^2=3\mu^2-2m^2$ and $m_2^2=\mu^2$. The dispersion relation $\omega_{1,2}(p)$ then reduces to

$$\omega_{1,2}(p) \;=\; \sqrt{p^2+3\mu^2-m^2\pm\sqrt{(3\mu^2-m^2)^2+4\mu^2p^2}}\,, \tag{16}$$

From this equation, we note that $\omega_2(p)$ is a massless mode. Expanding around zero momentum p, we find

$$\omega_2(p) \;=\; \sqrt{\frac{\mu^2-m^2}{3\mu^2-m^2}}\,p+\mathcal{O}(p^2)\,. \tag{17}$$

The mode is linear in the momentum for small momenta and is thus a conventional Goldstone mode. This is in agreement with the fact that the $O(2)$-symmetry has been broken due to the condensate ϕ_0.

A. Effective Action and Gap Equations

The 2PI effective action can be written as

$$\begin{aligned}\Omega[\phi_0,D] \;=\; & \frac{1}{2}\left(m^2-\mu^2\right)\phi_0^2+\frac{\lambda}{8N}\phi_0^4+\frac{1}{2}\mathrm{Tr}\ln D^{-1}\\ & +\frac{1}{2}\mathrm{Tr}D_0^{-1}D+\Phi[D]\,,\end{aligned} \tag{18}$$

where D is the exact propagator and $\Phi[D]$ is the sum of all two-particle irreducible vacuum diagrams. The trace is over field indices as well as over space-time. In the Hartree approximation one includes the double bubble diagrams shown in Fig. 1. If we denote by D_{ij} the components of the propagator D, the functional $\Phi[D]$ can be written as

$$\Phi[D] \;=\; \frac{\lambda}{8N}F_{ijkl}\sum\!\!\!\!\!\!\int_P D_{ij}\sum\!\!\!\!\!\!\int_Q D_{kl}\,, \tag{19}$$

where $F_{ijkl}=(\delta_{ij}\delta_{kl}+\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})$ is the sum of the three rank-four invariants of $O(N)$. The sum-integral above is defined by

$$\sum\!\!\!\!\!\!\int_Q \;\equiv\; \left(\frac{e^{\gamma_E}\Lambda^2}{4\pi}\right)^{\epsilon} T\sum_{q_0=2\pi nT}\int\frac{d^dq}{(2\pi)^d}\,, \tag{20}$$

where $Q=(q_0,\mathbf{q})$, $d=3-2\epsilon$ and Λ is the renormalization scale associated with dimensional regularization. The sum is over Matsubara frequencies. The integral over three-momentum q is calculated with dimensional regularization. We also introduce the compact notation for these integrals

$$\int_q \;\equiv\; \int\frac{d^dq}{(2\pi)^d}\,. \tag{21}$$

Generally, in the vacuum, the terms in the 2PI effective action (18) can be classified according to which order in the $1/N$-expansion they contribute and can be expressed in terms of $O(N)$ invariants such as $\mathrm{Tr}(D^n)$ and $\mathrm{Tr}(\phi_0^2D^n)$ [45]. The first and second term in Eq. (18) are proportional to $\mathrm{Tr}[\phi_0^2]$ and $[\mathrm{Tr}(\phi_0^2)]^2$, respectively. Each trace gives a factor of N and so they both scale as N. The third term is the trace of the full propagator and therefore scales as N. The third term can be decomposed into a sum of the terms $\mathrm{Tr}[D]$, $\mathrm{Tr}[\phi_0^2D]$, $\mathrm{Tr}[\phi_0^2]\mathrm{Tr}[D]$, and $\mathrm{Tr}[\phi_0^2D]$. Finally, let us consider $\Phi[D]$. Performing the sums involving F_{ijkl}, $\Phi[D]$ can be expressed in terms of the invariants $\mathrm{Tr}(D^2)$ and $[\mathrm{Tr}(D)]^2$:

$$\Phi[D] \;=\; \frac{\lambda}{8N}\left[\left[\mathrm{Tr}\,(D)\right]^2+2\mathrm{Tr}\left(D^2\right)\right]. \tag{22}$$

The first term involves two traces each giving a factor of N. There is a factor of $1/N$ coming from the vertex and so this term goes like N. Similarly, the second term involves one trace and one vertex and it contributes at order one. We therefore conclude that the Hartree approximation is not a systematic approximation. It is not systematic in powers of $1/N$ as we have seen and it is not systematic in number of loops since we have not included the setting-sun diagrams that arise in the broken phase.

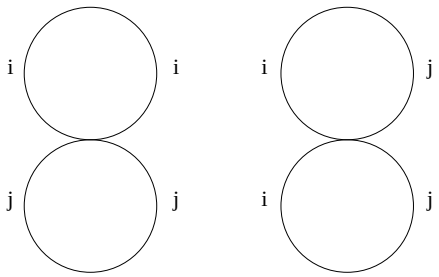

FIG. 1: Vacuum diagram contributing to the effective potential.in the Hartree approximation. Left diagrams are of order N, while right diagrams are of order one.

At the stationary points, the 2PI effective action satisfies the gap equations

$$\frac{\delta\Omega[\phi_0,D]}{\delta D} \;=\; 0\,, \tag{23}$$

$$\frac{\delta\Omega[\phi_0,D]}{\delta\phi_0} \;=\; 0\,. \tag{24}$$

The gap equation (23) can be rewritten as

$$D^{-1}=D_0^{-1}+2\frac{\delta\Phi[D]}{\delta D}\,. \tag{25}$$

Using the fact that $D^{-1}-D_0^{-1}=\Pi(P)$, where $\Pi(P)$ is the self-energy, we obtain

$$\Pi(P) \quad =2\frac{\delta\Phi[D]}{\delta D}\,. \tag{26}$$

Note that the self-energy in Eq. (26) is a matrix. The self-energy is obtained by cutting a line in the vacuum graphs. A generic loop diagram that contributes in the Hartree approximation is shown in Fig. 2.

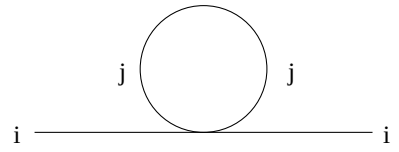

FIG. 2: Feynman diagram contributing to the self-energy $\Pi(P)$ in the Hartree approximation.

In the Hartree approximation, the self-energies are momentum independent. Moreover, it can be shown [14] that only the diagonal elements of the self-energy matrix Π_{ij} are nonzero. If we denote these elements by Π_i, the dressed inverse propagator is given by Eq. (9) with the tree level masses $m_1,\ m_2,\ m_3$ replaced by the effective medium-dependent masses $M_1,\ M_2,\ M_3$, where

$$M_i^2 \ = \ m_i^2 + \Pi_i \ . \tag{27}$$

The full propagator then reads

$$D(\omega_n,p) \ = \ \begin{pmatrix} \frac{\omega_n^2+p^2+M_1^2-\mu^2}{(\omega_n^2+\tilde{\omega}_1^2)(\omega_n^2+\tilde{\omega}_2^2)} & \frac{2\mu\omega_n}{(\omega_n^2+\tilde{\omega}_1^2)(\omega_n^2+\tilde{\omega}_2^2)} & 0 & 0 & \dots \\ \frac{-2\mu\omega_n}{(\omega_n^2+\tilde{\omega}_1^2)(\omega_n^2+\tilde{\omega}_2^2)} & \frac{\omega_n^2+p^2+M_2^2-\mu^2}{(\omega_n^2+\tilde{\omega}_1^2)(\omega_n^2+\tilde{\omega}_2^2)} & 0 & 0 & \dots \\ 0 & 0 & \frac{1}{\omega_n^2+\tilde{\omega}_3^2} & 0 & \dots \\ 0 & 0 & 0 & \frac{1}{\omega_n^2+\tilde{\omega}_3^2} & \dots \\ \dots & \dots & \dots & \dots & \dots \end{pmatrix} , \tag{28}$$

where $\tilde{\omega}_{1,2,3}(p)$ are obtained from Eqs. (13) and (14) by the replacement $m_i \to M_i$. The functional $\Phi[D]$ can then be written out explcitly as

$$\begin{aligned} \Phi[D] \ = \ \frac{\lambda}{8N}\Big[& 3\sum\!\!\!\!\!\!\int_Q D_{11}\sum\!\!\!\!\!\!\int_K D_{11} + 3\sum\!\!\!\!\!\!\int_Q D_{22}\sum\!\!\!\!\!\!\int_K D_{22} + 2\sum\!\!\!\!\!\!\int_Q D_{11}\sum\!\!\!\!\!\!\int_K D_{22} + 4(N-1)\sum\!\!\!\!\!\!\int_Q D_{11}\sum\!\!\!\!\!\!\int_K D_{33} \\ & +4(N-1)\sum\!\!\!\!\!\!\int_Q D_{22}\sum\!\!\!\!\!\!\int_K D_{33} + 4N(N-1)\sum\!\!\!\!\!\!\int_Q D_{33}\sum\!\!\!\!\!\!\int_K D_{33}\Big] \ . \end{aligned} \tag{29}$$

Notice that the terms involving off-diagonal elements of the full propagator D are absent. This is due to the fact the diagram vanishes upon summation over the Matsubara frequencies. This follows immediately from Eq. (28).

The gap equations for the dressed masses now follow from Eqs. (25), (26), (27), and (29). These equations contain ultraviolet divergences and require renormalization. Renormalization is discussed in Appendix A and the diagrammatic interpretation of the iterative renormalization procedure is shown in Fig. 3.

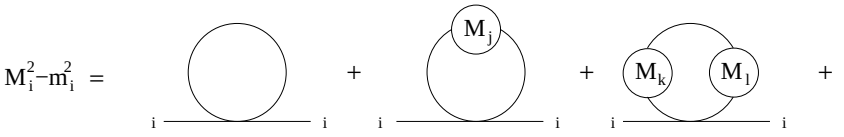

FIG. 3: Diagrammatic interpretation of the gap equations for m_i in the 2PI Hartree approximation.

After renormalization, we obtain

$$M_1^2 - m_1^2 \ = \ \frac{\lambda}{2N}\left[3(J_1^c + J_1^T) + (J_2^c + J_2^T) + (2N-2)(J_3^c + J_3^T)\right] \ , \tag{30}$$

$$M_2^2 - m_2^2 \ = \ \frac{\lambda}{2N}\left[(J_1^c + J_1^T) + 3(J_2^c + J_2^T) + (2N-2)(J_3^c + J_3^T)\right] \ , \tag{31}$$

$$M_3^2 - m_3^2 \ = \ \frac{\lambda}{2N}\left[(J_1^c + J_1^T) + (J_2^c + J_2^T) + 2N(J_3^c + J_3^T)\right] \ , \tag{32}$$

where the integrals J_n^c and J_n^T are defined in Appendix A.

We next consider the gap equation (24). The expression for it follows from (23) and (29). After renormalization, we obtain

$$0 \;=\; \phi_0 \left[m^2 - \mu^2 + \frac{\lambda}{2N}\phi_0^2 + \frac{\lambda}{2N}\left[3(J_1^c + J_1^T) + (J_2^c + J_2^T) + (2N-2)(J_3^c + J_3^T) \right] \right] . \tag{33}$$

Using the gap equation (30) to eliminate the integrals J_n^c and J_n^T, we can rewrite Eq. (33) in a very simple way

$$0 \;=\; \phi_0 \left[M_1^2 - \mu^2 - \frac{\lambda}{N}\phi_0^2 \right] . \tag{34}$$

The difference between this equation and the corresponding equation at the tree-level, is the replacements $m_1^2 \to M_1^2$. Similarly, combining the gap equations (31) and (33), we obtain

$$M_2^2 \;=\; \mu^2 + \frac{\lambda}{N}\left[(J_2^c + J_2^T) - (J_1^c + J_1^T) \right] . \tag{35}$$

Comparing this equation with the tree-level result $m_2^2 = \mu^2$, Eq. (35), we see that the Goldstone theorem is not respected. It is well known that the 2PI Hartree approximation violates Goldstone's theorem and in the present case this means that there is no massless mode associated with the breaking of the $O(2)$-symmetry due to the condensate ϕ_0. Writing $M_2^2 = \mu^2 + \delta$, where $\delta = \frac{\lambda}{N}\left[(J_2^c + J_2^T) - (J_1^c + J_1^T) \right]$, a calculation analogous to the one leading to Eq. (17), shows that the mass gap of the Goldstone mode is given by

$$\Delta M^2_{\rm GB} \;=\; \frac{M_1^2 - m^2}{3\mu^2 + M_1^2}\delta . \tag{36}$$

In the 2PI $1/N$-expansion, the Goldstone theorem is satisfied order by order. In the large-N limit, it is clear from Eq. (35) that the Goldstone theorem is respected since the last term vanishes.

The effective potential (18) follows from Eqs. (9), (28) and (29) It can be renormalized in the same manner as the gap equations and details can be found in Appendix A. After renormalization, we obtain

$$\begin{aligned}
\Omega \;=\; & \frac{1}{2}(m^2-\mu^2)\phi_0^2 + \frac{\lambda}{8N}\phi_0^4 + \frac{1}{2}(\mathcal{J}_1^c + \mathcal{J}_1^T) + \frac{1}{2}(\mathcal{J}_2^c + \mathcal{J}_2^T) + (N-1)(\mathcal{J}_3^c + \mathcal{J}_3^T) \\
& - \frac{1}{2}(M_1^2 - m_1^2)(J_1^c + J_1^T) - \frac{1}{2}(M_2^2 - m_2^2)(J_2^c + J_2^T) - (N-1)(M_3^2 - m_3^2)(J_3^c + J_3^T) \\
& + \frac{3\lambda}{8N}(J_1^c + J_1^T)^2 + \frac{3\lambda}{8N}(J_2^c + J_2^T)^2 + \frac{\lambda}{2}(N-1)(J_3^c + J_3^T)^2 + \frac{\lambda}{4N}(J_1^c + J_1^T)(J_2^c + J_2^T) \\
& + \frac{\lambda}{2N}(N-1)(J_1^c + J_1^T)(J_3^c + J_3^T) + \frac{\lambda}{2N}(N-1)(J_2^c + J_2^T)(J_3^c + J_3^T) ,
\end{aligned} \tag{37}$$

where $\mathcal{J}_n^c$ and $\mathcal{J}_n^T$ are defined in Appendix A.

III. KAONS IN THE CFL PHASE

A. Effective theories

In the CFL phase of dense QCD, the original symmetry $SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B$ is broken down to $SU(3)_{c+L+R}$. The diquark condensate $\langle \psi\psi \rangle$ breaks chiral symmetry in exactly the same manner as in vacuum QCD and so the effective Lagrangian for the Goldstone modes have the same structure as in chiral perturbation theory. Notice, however, that the mesons are composed of four quark fields of the form $\bar\psi\bar\psi\psi\psi$ instead of the conventional $\bar\psi\psi$. Nevertheless, one finds the same quantum numbers. Another difference is that Lorentz invariance is broken due to the presence of the chemical potentials and so the Lagrangian in invariant only under rotations in three dimensions. The chiral effective Lagrangian is given by [10]

$$\begin{aligned}
\mathcal{L} \;=\; & \frac{1}{4}f_\pi^2 \mathrm{Tr}\left[(\partial_0\Sigma + i[A,\Sigma])\left(\partial_0\Sigma - i[A,\Sigma]^\dagger\right) \right. \\
& \left. - v_\pi^2 (\partial_i \Sigma)(\partial_i \Sigma^\dagger) \right] \\
& + \frac{1}{2} a f_\pi^2 \det M \mathrm{Tr}[M^{-1}(\Sigma + \Sigma^\dagger)] \; + \cdots ,
\end{aligned} \tag{38}$$

where f_π, v_π, and a are constants. The meson field Σ is given by

$$\Sigma \;=\; e^{i\lambda^a\phi^a/f_\pi}\;, \tag{39}$$

where λ^a are the Gell-Mann matrices and ϕ^a describe the octet of Goldstone bosons. The matrix A acts as the zeroth component of a gauge field and is given by

$$A \;=\; \mu_Q Q - \frac{M^2}{2\mu}\;, \tag{40}$$

where μ_Q is the chemical potential for electric charge, μ is the baryon chemical potential, $Q = \mathrm{diag}(2/3, -1/3, -1/3)$, and $M = \mathrm{diag}(m_u, m_d, m_s)$.

At asymptotically high densities, one can use perturbative QCD calculations to determine the parameters f_π, v_π, and a by matching [10, 12, 13].

$$f_\pi^2 \;=\; \frac{12-8\ln 2}{18\pi^2}\mu^2\;, \tag{41}$$

$$v_\pi^2 \;=\; \frac{1}{3}\;, \tag{42}$$

$$a \;=\; \frac{3\Delta^2}{\pi^2 f_\pi^2}\;, \tag{43}$$

where Δ is the superconducting gap. Note that $v_\pi = 1/\sqrt{3}$ is the standard result for the speed of sound in a dense medium. In the vacuum, Lorentz invariance enforces the value $v_\pi = 1$. It is important to stress that the values of these parameters given by Eqs. (41)–(43) are valid only at asympotically high densites. For moderate densities, which are relevant for compact stars, one does not know the values of these parameters as they cannot be determined by matching.

After expanding to fourth order in the meson fields, Alford, Braby, and Schmitt obtain the following effective Lagrangian for the kaons:

$$\begin{aligned}\mathcal{L} \;=\; & \left[(\partial_0+\mu_1)\Phi_1^\dagger\right]\left[(\partial_0-\mu_1)\Phi_1\right] + (\partial_i\Phi_1^\dagger)(\partial_i\Phi_1) + \left[(\partial_0+\mu_2)\Phi_2^\dagger\right]\left[(\partial_0-\mu_2)\Phi_2\right] + \frac{1}{2}m_1^2\Phi_1^2 \\ & + \frac{1}{2}m_2^2\Phi_2^2 + \frac{\beta_1}{4}\Phi_1^4 + \frac{\beta_2}{4}\Phi_2^4 + \frac{\alpha}{2}\Phi_1^2\Phi_2^2\;,\end{aligned} \tag{44}$$

where the complex doublet are $(K^+, K^0) = (\Phi_1, \Phi_2)$ and the parameters are given by

$$m_1^2 \;=\; am_d(m_s+m_u) \tag{45}$$

$$m_2^2 \;=\; am_u(m_s+m_d) \tag{46}$$

$$\mu_1 \;=\; \mu_Q + \frac{m_s^2-m_u^2}{2\mu} \tag{47}$$

$$\mu_2 \;=\; \frac{m_s^2-m_d^2}{2\mu} \tag{48}$$

$$\beta_i \;=\; \frac{1}{6f_\pi^2}\left(4\mu_i^2 - m_i^2\right)\;, \tag{49}$$

$$\alpha \;=\; \frac{1}{2}\left(\beta_1+\beta_2\right) - \left(\frac{\mu_1-\mu_2}{2f_\pi}\right)^2\;. \tag{50}$$

The dimensionless parameters α and β_i are quartic couplings of the effective theory. Note in particular that they depend on the chemical potentials μ_i. The effective theory described by the Lagrangian (44) is invariant under the group $O(2)\times O(2)$.

There is a technical complication arising from using the Lagrangian Eq. (44). The problem is that the effective couplings depend on the chemical potentials. This implies that the counterterms also depend on the chemical potentials i.e. parameters that describe a dense medium. Renormalizing a theory based on the Lagrangian (44) therefore depends on the medium, which one may object to. We therefore take a somewhat different approach by using an effective Lagrangian with mass parameters and couplings that are independent of the chemical potentials. For simplicity, we set $v_\pi = 1$, but it is not difficult to scale loop momenta in our equations to take into account values of v_π that differ from the its vacuum value. The kaons are written as a complex doublet [1], $(K^0, K^+) = (\Phi_1, \Phi_2)$. The Euclidean Lagrangian with an $O(2)\times O(2)$ symmetry is given by

$$\begin{aligned}\mathcal{L} \;=\; & \left[(\partial_0+\mu_0)\Phi_1^\dagger\right]\left[(\partial_0-\mu_0)\Phi_1\right] + (\partial_i\Phi_1^\dagger)(\partial_i\Phi_1) \\ & + \left[(\partial_0+\mu_+)\Phi_2^\dagger\right]\left[(\partial_0-\mu_+)\Phi_2\right] + (\partial_i\Phi_2^\dagger)(\partial_i\Phi_2) \\ & + m_0^2\Phi_1^\dagger\Phi_1 + m_+^2\Phi_2^\dagger\Phi_2 + \frac{\lambda_0}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 \\ & + \frac{\lambda_+}{2}\left(\Phi_2^\dagger\Phi_2\right)^2 + \lambda_H\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right)\;.\end{aligned} \tag{51}$$

The chemical potentials μ_0 and μ_+ associated with the two conserved charges for each complex field Φ_i. They

[1] Note that the identification in Ref. [14] is $(K^+, K^0) = (\Phi_1, \Phi_2)$.

are related to the quark chemical potentials μ_u, μ_d, and μ_s by

$$\mu_0 = \mu_d - \mu_s \,, \tag{52}$$
$$\mu_+ = \mu_u - \mu_s \,. \tag{53}$$

We therefore have $\mu_+ - \mu_0 = \mu_u - \mu_d$. In weak equilibrium, the processes $d+\nu \leftrightarrow u+e^-$ go with the same rate in both directions. If we assume that the neutrinos leave the system, their chemical potential is $\mu_\nu = 0$. This implies that $\mu_d = \mu_u - \mu_Q$, where μ_Q is the electric charge chemical potential. In other words, $\mu_Q = \mu_+ - \mu_0$. In the absence of the chemical potentials and with $m_0 = m_+$, and $\lambda_0 = \lambda_+ = \lambda_H$, the Lagrangian (51) has an extended $SO(4) \sim SU(2)_L \times SU(2)_R$ symmetry. If we add chemical potentials such that $\mu_0 = \mu_+$, this symmetry is broken down to $SU(2) \times U(1)$, where the chemical potential is for the $U(1)$ charge. A condensate would then break this symmetry down to $U(1)$ implying the existence of massless modes. Naively, one would perhaps expect three Goldstone modes as there are three broken generators. However, one of the massless mode is quadratic in the momentum p for small p and the Nielsen-Chadha theorem implies that such a mode be counted twice [25, 48, 49]. This is consistent with the fact that there are only two massless modes.

B. Effective potential and gap equations

In order to allow for a condensate of neutral kaons, we introduce an expectation value ϕ_0 for Φ_1 and write

$$\Phi_1 = \frac{1}{\sqrt{2}} \left(\phi_0 + \phi_1 + i\phi_2\right) \,, \tag{54}$$

where ϕ_1 and ϕ_2 are quantum fluctuting fields.

The inverse tree-level propagator can be written as a block-diagonal 4×4 matrix:

$$D_0^{-1}(\omega_n, p) = \begin{pmatrix} \omega_n^2 + p^2 + m_1^2 - \mu_0^2 & -2\mu_0\omega_n & 0 & 0 \\ 2\mu_0\omega_n & \omega_n^2 + p^2 + m_2^2 - \mu_0^2 & 0 & 0 \\ 0 & 0 & \omega_n^2 + p^2 + m_3^2 - \mu_+^2 & -2\mu_+\omega_n \\ 0 & 0 & 2\mu_+\omega_n & \omega_n^2 + p^2 + m_3^2 - \mu_+^2 \end{pmatrix} \,, \tag{55}$$

where the tree-level masses are

$$m_1^2 = m_0^2 + \frac{3\lambda_0}{2}\phi_0^2 \,, \tag{56}$$
$$m_2^2 = m_0^2 + \frac{\lambda_0}{2}\phi_0^2 \,, \tag{57}$$
$$m_3^2 = m_+^2 + \frac{\lambda_H}{2}\phi_0^2 \,. \tag{58}$$

Note that in the remainder of this section, the mass parameters m_0^2 and m_+^2 are positive. The classical potential is

$$V = \frac{1}{2}\left(m_0^2 - \mu_0^2\right)\phi_0^2 + \frac{\lambda_0}{8}\phi_0^4 \,. \tag{59}$$

The dispersion relations are

$$\omega_{1,2}(p) = \sqrt{p^2 + \frac{1}{2}(m_1^2 + m_2^2) + \mu_0^2 \pm \sqrt{4\mu_0^2\left[p^2 + \frac{1}{2}(m_1^2 + m_2^2)\right] + \frac{1}{4}\left(m_1^2 - m_2^2\right)^2}} \,, \tag{60}$$
$$\omega_{3,4}(p) = \sqrt{p^2 + m_3^2} \pm \mu_+ \,. \tag{61}$$

The 2PI effective action is given by

$$\Omega[\phi_0, D] = \frac{1}{2}\left(m^2 - \mu_0^2\right)\phi_0^2 + \frac{\lambda}{8N}\phi_0^4 + \frac{1}{2}\mathrm{Tr}\ln D^{-1} + \frac{1}{2}\mathrm{Tr}D_0^{-1}D + \Phi[D] \,, \tag{62}$$

where $\Phi[D]$ contains all 2PI vacuum diagrams. In the Hartree approximation, we include all double-bubble diagrams which can be written in terms of $O(2) \times O(2)$ invariants. If we denote by D_a and D_b the two 2×2 submatrices of the propagator D, we can write [42]

$$\Phi[D] = \frac{\lambda_0}{8}\left[\mathrm{Tr}\left(D_a\right)\right]^2 + 2\mathrm{Tr}\left(D_a^2\right)\Big] + \frac{\lambda_+}{8}\left[\mathrm{Tr}\left(D_b\right)\right]^2 + 2\mathrm{Tr}\left(D_b^2\right)\Big] + \frac{\lambda_H}{4}(\mathrm{Tr}D_a)(\mathrm{Tr}D_b) \,. \tag{63}$$

Writing out explicitly the terms in $\Phi[D]$, we find

$$\begin{aligned}\Phi[D] &= \frac{3}{8}\left[\lambda_0 \sum\!\!\!\!\!\!\int_K D_{11} \sum\!\!\!\!\!\!\int_Q D_{11} + \lambda_0 \sum\!\!\!\!\!\!\int_K D_{22} \sum\!\!\!\!\!\!\int_Q D_{22} + \lambda_+ \sum\!\!\!\!\!\!\int_K D_{33} \sum\!\!\!\!\!\!\int_Q D_{33} + \lambda_+ \sum\!\!\!\!\!\!\int_K D_{44} \sum\!\!\!\!\!\!\int_Q D_{44}\right] \\ &+\frac{1}{4}\left[\lambda_0 \sum\!\!\!\!\!\!\int_K D_{11} \sum\!\!\!\!\!\!\int_Q D_{22} + \lambda_+ \sum\!\!\!\!\!\!\int_K D_{33} \sum\!\!\!\!\!\!\int_Q D_{44} + \lambda_H \sum\!\!\!\!\!\!\int_K D_{11} \sum\!\!\!\!\!\!\int_Q D_{33} + \lambda_H \sum\!\!\!\!\!\!\int_K D_{11} \sum\!\!\!\!\!\!\int_Q D_{44}\right. \\ &\left.+\lambda_H \sum\!\!\!\!\!\!\int_K D_{22} \sum\!\!\!\!\!\!\int_Q D_{33} + \lambda_H \sum\!\!\!\!\!\!\int_K D_{22} \sum\!\!\!\!\!\!\int_Q D_{44}\right] . \end{aligned} \tag{64}$$

Again the terms involving the off-diagonal elements of the full propagator D are absent since they vanish upon summation over the Matsubara frequencies.

The gap equations now follow in the usual manner. They require renormalization, which is briefly discussed in Appendix A. After renormalization, the gap equations read

$$M_1^2 - m_1^2 = \frac{1}{2}\left[3\lambda_0(J_1^c + J_1^T) + \lambda_0(J_2^c + J_2^T) + 2\lambda_H(J_3^c + J_3^T)\right] , \tag{65}$$

$$M_2^2 - m_2^2 = \frac{1}{2}\left[\lambda_0(J_1^c + J_1^T) + 3\lambda_0(J_2^c + J_2^T) + 2\lambda_H(J_3^c + J_3^T)\right] , \tag{66}$$

$$M_3^2 - m_3^2 = \frac{1}{2}\left[\lambda_H(J_1^c + J_1^T) + \lambda_H(J_2^c + J_2^T) + 4\lambda_+(J_3^c + J_3^T)\right] , \tag{67}$$

$$0 = \phi_0\left[m_0^2 - \mu_0^2 + \frac{\lambda_0}{2}\phi_0^2 + \frac{1}{2}\left[3\lambda_0(J_1^c + J_1^T) + \lambda_0(J_2^c + J_2^T) + 2\lambda_H(J_3^c + J_3^T)\right]\right] , \tag{68}$$

In the appendix we argue that M_4 is equal to M_3 and their gap equations are idential. This is only correct as long as there is no charged kaon condensate. Substituting Eq. (65) into Eq. (68), it can be written as

$$\phi_0\left[M_1^2 - (\mu_0^2 + \lambda_0\phi_0^2)\right] = 0 , \tag{69}$$

Similarly, substituting Eq. (68) into Eq. (66), we obtain

$$M_2^2 - \mu_0^2 = \lambda_0\left[(J_2^c + J_2^T) - (J_1^c + J_1^T)\right] . \tag{70}$$

The effective potential can be renormalized using the same methods and after renormalization the effective potential is given by

$$\begin{aligned}\Omega &= \frac{1}{2}(m_0^2 - \mu_0^2)\phi_0^2 + \frac{\lambda_0}{8}\phi_0^4 + \frac{1}{2}\left(\mathcal{J}_1^c + \mathcal{J}_1^T\right) + \frac{1}{2}\left(\mathcal{J}_2^c + \mathcal{J}_2^T\right) + \left(\mathcal{J}_3^c + \mathcal{J}_3^T\right) \\ &-\frac{1}{2}(M_1^2 - m_1^2)(J_1^c + J_1^T) - \frac{1}{2}(M_2^2 - m_2^2)(J_2^c + J_2^T) - (M_3^2 - m_3^2)(J_3^c + J_3^T) \\ &+\frac{3\lambda_0}{8}\left(J_1^c + J_1^T\right)^2 + \frac{3\lambda_0}{8}\left(J_2^c + J_2^T\right)^2 + \lambda_+\left(J_3^c + J_3^T\right)^2 + \frac{\lambda_0}{4}\left(J_1^c + J_1^T\right)\left(J_2^c + J_2^T\right) \\ &+\frac{\lambda_H}{2}\left(J_1^c + J_1^T\right)\left(J_2^c + J_2^T\right) + \frac{\lambda_H}{2}\left(J_2^c + J_2^T\right)\left(J_3^c + J_3^T\right) . \end{aligned} \tag{71}$$

C. Phase diagram and quasiparticle masses

In order to determine the phase diagram and numerical evaluate the masses of the quasiparticle, we need to know the values of the parameters in the effective Lagrangian (51). In Ref. [14], the authors use a quark chemical potential $\mu \simeq 500$ MeV and and superconducting gap $\Delta \simeq 30$ MeV. Extrapolating from asymptotically high densities, Eq. (41), and Eqs. (45)–(48) give $f_\pi \simeq 100$ MeV, $\mu_i \simeq 20$ MeV, $m_1 \simeq 5$ MeV, and $m_2 \simeq 4$ MeV. In the numerical calculatations that we present below, we use $m_0 = 4$ MeV, $m_+ = 5$ MeV, $\mu_0 = \mu_+ = 4.5$ MeV, and $f_\pi = 100$ MeV unless otherwise stated. The renormalization scale is chosen to be the average of the two vacuum masses, i.e. $\Lambda = 4.5$ MeV. Using Eqs.(49)–(50) and identifying $2\beta_1$ with λ_0, $2\beta_2$ with λ_+, 2α with λ_H, the numerical values are $\lambda_0 = 1.25 \times 10^{-3}$, $\lambda_+ = 1, .08 \times 10^{-3}$, and $\lambda_H = 1.16 \times 10^{-3}$. In the plots, where we vary the chemical potentials, we use the same

couplings throughout as μ-dependent coupling constants are problematic.

The neutral kaons K^0 and $\bar{K^0}$ are identified with the linear combinations of the fields ϕ_1 and ϕ_2, and therefore with $\tilde{\omega}_1(p)$ and $\tilde{\omega}_2(p)$. Similarly, the charged kaons K^- and K^+ are given by linear combinations of ϕ_3 and ϕ_4 and are identified with $\tilde{\omega}_3(p)$ and $\tilde{\omega}_4(p)$.

In Fig. 4, we show the neutral kaon condensate as a function of μ_0 and μ_+ for $T = 0$. For $\mu_+ = 0$, i.e. along the μ_0-axis, there is a second-order phase transition to a neutral phase with a kaon condensate at a critical chemical potential $\mu_0 = m_2$. This is the CFL-K^0 phase. For larger values of μ_+ the transition becomes first order. The point in the (μ_0, μ_+)-plane where the transition changes order is a critical point and given by $(4.0, 5.0)$ MeV. In the part of the phase diagram where the transition is first order, the transition is actually to a phase with condensate of charged kaons. This condensate is not shown in the figure. This is the CFL-K^+ phase. Thus there is a competition between the neutral and the charged condensates and nowhere do they exist simultaneously. The transitions to the kaon-condensed phases are density driven.

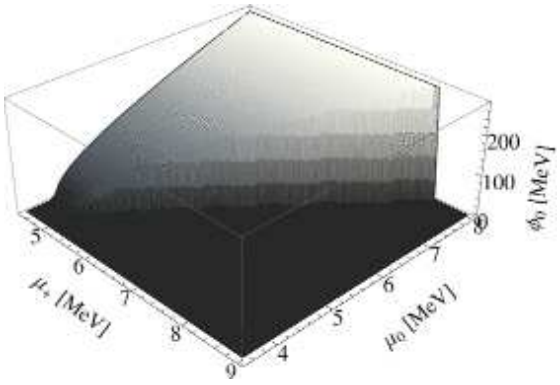

FIG. 4: Neutral kaon condensate as a function of the chemical potentials μ_0 and μ_+ for $T = 0$.

In Fig. 5, we show the neutral kaon condensate as a function of μ_0 and μ_+ for $T = 200$ MeV. The point in the (μ_0, μ_+)-plane where the transition changes order is given by $(5.2, 6.0)$ MeV.

In Fig. 6, we show the mass parameters $M_{1,2}$ and M_3 normalized to μ_0 for $\mu_0 = \mu_+ = 4.5$ MeV as a function of T normalized to T_c. The masses M_3 and M_4 are degenerate for all values of T, while M_1 and M_2 become degenerate at the critical temperarture. If the Goldstone theorem is obeyed M_2 is exactly equal to μ_0 in the broken phase. We notice that there is a tiny deviation.

In Fig 7, we show the thermodynamic potential $\Omega(\phi_0) - \Omega(0)$ as a function of the condensate ϕ_0 for $\mu_0 = \mu_+ = 4.5$ MeV and three different values of the temperature. $\Omega(\phi_0)$ is obtained by solving the gap equation for the masses (65), (66), and (68) and inserting the values into the effectifve potential (71). The solid line is $T = 0$, the dashed line is $T = T_c = 118.5$ MeV, and the dotted line is $T = 200$ MeV. The phase transition is second order.

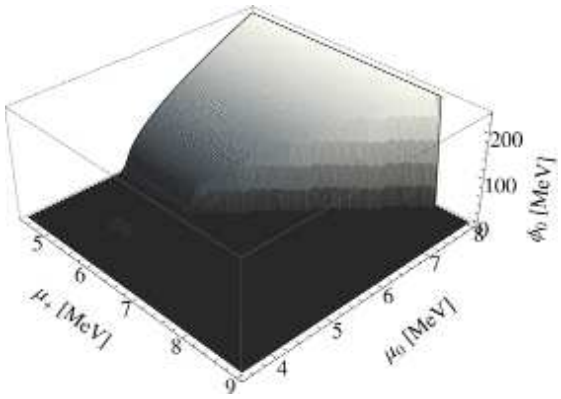

FIG. 5: Neutral kaon condensate as a function of the chemical potentials μ_0 and μ_+ for $T = 200$ MeV.

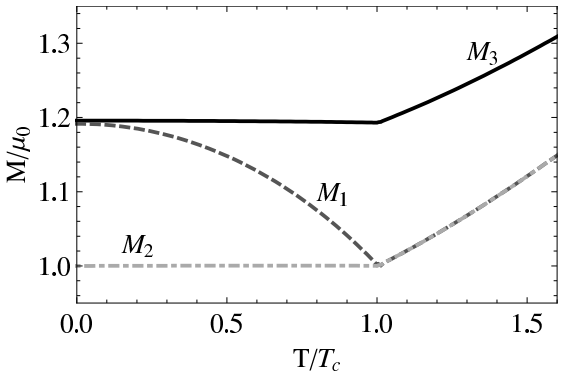

FIG. 6: Dressed masses $M_{1,2}$ and M_3 normalized to μ_0 as a function of T/T_c for $\mu_0 = \mu_+ = 4.5$ MeV.

In Fig. 8 we show the masses of K^0 ($\omega_2(q = 0)$) and K^+ ($\omega_4(q = 0)$) for $\mu_0 = \mu_+ = 4.5$ MeV and as functions of T normalized to T_c. We notice that the mass of K^0 is not strictly zero, which explicitly shows that the Goldstone theorem is not respected by the Hartree approximation [2]. In Ref. [14], the authors make some further approximations of the sum-integrals appearing in the gap equations. These approximations give rise to an exactly gapless mode. As pointed out in their paper and as can be seen in Fig. 8, this is a very good

[2] Note that the mass gap does vanish at $T = T_c$ since $J_1^c = J_2^c$ and $J_1^T = J_2^T$, cf. Eq (70).

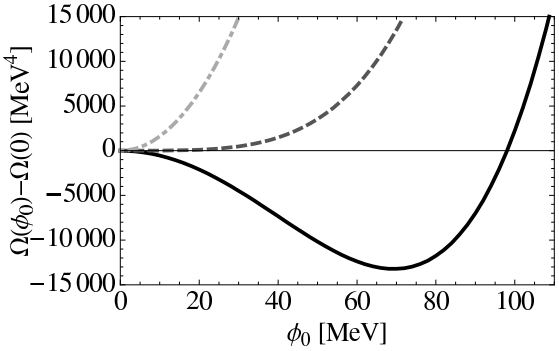

FIG. 7: $\Omega(\phi_0) - \Omega(0)$ for $\mu_0 = \mu_+ = 4.5$ MeV and three different values of the temperature. The solid line is $T = 0$, the dashed line is $T = T_c = 118.5$ MeV, and the dotted line is $T = 200$ MeV.

approximation.

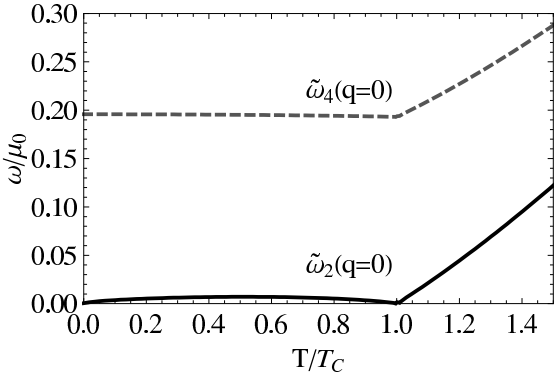

FIG. 8: Mass gaps for the K^+ and K^0 modes for $\mu_0 = \mu_+ = 4.5$ MeV and as a function of T normalized to T_c.

In Fig. 9, we show the difference $M_2^2 - \mu_0^2$, Eq. (70), normalized to μ_0^2 for $\mu_0 = 4.5$ MeV as a function of T/T_c. This is a another measure of the violation of Goldstone's theorem. We see that the violation is tiny.

D. Effects of charge neutrality

Bulk matter in compact stars must be overall color and electrically neutral, otherwise one pays an enormous energy penalty. The neutrality constraint applies whether or not the gauge charges are broken or not. For certain values of μ_0, μ_+, and T, the thermodynamically stable state, i.e. Ω evaluated at the stationary points, has an overall electric charge. For example, in the region of the lower corner of Fig. 4, there is a charged kaon condensate which is positively charged.

In this subsection, we impose the constraint of overall charge neutrality. In some case this constraint can have

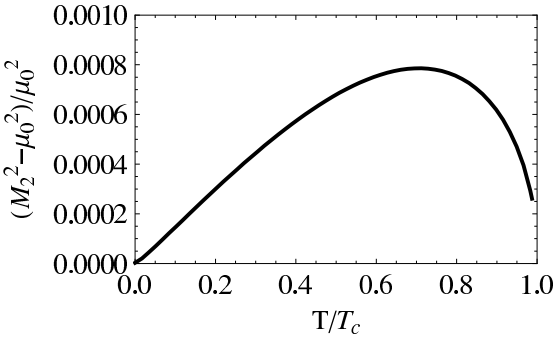

FIG. 9: Measure of the violation of Goldstone's theorem, $(M_2^2 - \mu_0^2)/\mu_0^2$ as a function of T/T_c.

dramatic effects on the phase diagram. For example, in a recent study by Abuki *et al* [50], they showed that pion condensation in the presence of finite isospin chemical potential does not occur for physical masses of the pions. In fact, they found a tiny window of pion condensation for pion masses below approximately 10 KeV. Thus pion condensation is very sensitive to the explicit chiral symmetry breaking.

For simplicity, we assume that we can describe the background of electrons by an ideal Fermi gas. We then add to the Lagrangian (51), the term

$$\mathcal{L}_{\rm electrons} \;=\; \tilde{\psi}_e \left(\gamma^\mu \partial_\mu + \gamma^0 \mu_e e - m_e\right) \psi_e \;, \qquad (72)$$

where ψ_e denotes the electron field, e is the electron charge, and m_e is the mass of the electron. The contribution to the free energy from the electrons is denoted by Ω_e and reads

$$\begin{aligned} \Omega_e \;&=\; -2 \int_p \Big\{ E_p + T \log\left[1 - e^{-\beta(E_p - \mu_Q)}\right] \\ &\quad + T \log\left[1 - e^{-\beta(E_p + \mu_Q)}\right] \Big\} \;. \end{aligned} \qquad (73)$$

where $E_p = \sqrt{p^2 + m_e^2}$. In the following, we assume the electrons are massless. If we use dimensional regularization, the first term in Eq. (73) vanishes since there is no mass scale. The temperature-dependent integrals can be done analytically and we obtain

$$\Omega_e \;=\; -\frac{1}{12\pi^2}\mu_Q^4 - \frac{1}{6}\mu_Q^2 T^2 - \frac{7\pi^2}{180} T^4 \;. \qquad (74)$$

We add Eq. (74) to Eq. (51) to obtain the full thermodynamic potential. The contribution n_+ to the electric charge density from the kaons is then given by

$$\begin{aligned} n_+ \;&=\; -\frac{\partial \Omega}{\partial \mu_Q} \\ &=\; -\frac{1}{2} \mathrm{Tr}\left[\frac{\partial D_0^{-1}}{\partial \mu_+} D\right] \;. \end{aligned} \qquad (75)$$

Using Eqs. (71), (74), and (75), we obtain

$$n_+ \;=\; 2\sum\!\!\!\!\!\!\int_Q \frac{\mu_+(-\omega_n^2+p^2+M_3^2-\mu_+^2)}{[\omega_n^2+\tilde{\omega}_3^2(q)][\omega_n^2+\tilde{\omega}_4^2(q)]}\;. \qquad (76)$$

This expression is free of ultraviolet divergences. After summing over Matsubara frequencies and averaging over angles, the equation reduces to

$$n_+ \;=\; \frac{1}{4\pi^2}\int_0^\infty \left[\frac{1}{e^{\tilde{\omega}_3(q)/T}-1}-\frac{1}{e^{\tilde{\omega}_4(q)/T}-1}\right] q^2\,dq\;. \qquad (77)$$

The contribution n_e to the electric charge density from the electrons is given by

$$\begin{aligned} n_e \;&=\; -\frac{\partial\Omega_e}{\partial\mu_Q} \\ &=\; -\frac{1}{3\pi^2}\mu_Q^3-\frac{1}{3}\mu_Q T^2\;. \end{aligned} \qquad (78)$$

Charge neutrality amounts to requiring that

$$n_+ + n_e \;=\; 0\;. \qquad (79)$$

In the case where we do not impose the charge neutrality requirement (79), we have two chemical potentials that we can vary freely. The charge neutrality constraint gives a relation between μ_0 and μ_+ and only one of them is free to vary. For a given value of T and e.g. μ_0, we must therefore solve simultaneously the gap equations (65)–(70) and the equation (79), to find the dressed masses M_i, ϕ_0 and the chemical potential μ_+. In Fig. 10, we show the neutral kaon condensate for neutral matter as a function of T and μ_0 For $T=0$, the onset of kaon condensation is at $\mu_0 = m_0$ as expected. The transition is second order for all values of μ_0 and again the transition is to a symmetric phase.

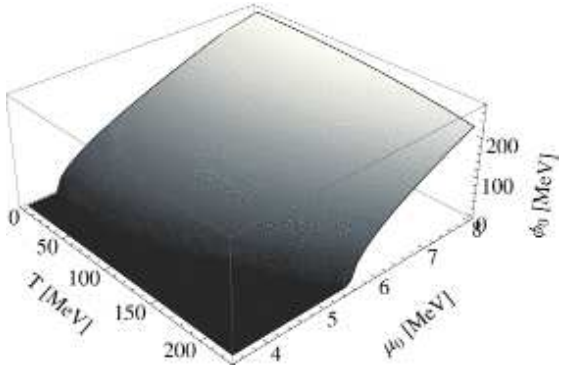

FIG. 10: The neutral kaon condensate as a function of T and μ_0 for a system which is electrically neutral.

In Fig. 11, we show μ_+ and $-\mu_Q = \mu_0 - \mu_+$ normalized to μ_0 as functions of temperature T. The sum of the curves are always equal to one. The chemical potential $-\mu_Q$ vanishes at $T=0$, increases rapidly, and is essentially constant for $T \geq 20$ MeV. The chemical potential μ_+, which equals μ_0 at $T=0$, never reaches its critical value $\mu_+^c = m_+$ and this explains why the phase transition in Fig. 10 is to a symmetric phase and not to a charged kaon condensed phase. In other words, charge neutrality does not allow for a charged condensate.

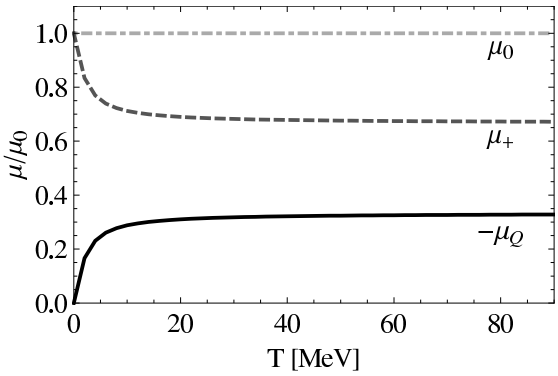

FIG. 11: The chemical potentials μ_+ and $-\mu_Q$ divided by μ_0 as functions of temperature in an electrically neutral system.

Finally we compare the critical temperature with and without neutrality. In the case with no neutrality constraint, the critical temperature was 118.5 MeV for $\mu_0 = \mu_+ = 4.5$ MeV. In the neutral case, μ_+ is a function of μ_0 and so varies with temperature. At $T=0$, we always have $\mu_Q = 0$, i. e. $\mu_0 = \mu_+$. We therefore choose the same value, i.e. $\mu_0 = 4.5$ MeV. The critical temperaature in this case is $T_c = 125$ MeV. In other words, there is a small increase in the critical temperature as one impose charge neutrality. This is in agreement with the findings of Ref. [14].

IV. SUMMARY AND OUTLOOK

In the present paper, we have studied the $O(N)$ and $O(2)\times O(2)$ models with chemical potentials using the 2PI formalism in the Hartree approximation. We have explicitly shown it is possible to renormalize the gap equation and effective potential in a way that is independent of temperature and chemical potentials. We have studied the phase diagram and the quasiparticle masses of the $O(2)\times O(2)$ model where the neutral kaons condense at sufficiently low temperature and sufficiently large value of the chemical potential. If the transition is first order, it turns out that the transition is not to a symmetric state but to a state with a K^+ condensate. This is in agreement with the findings of Alford, Braby, and Schmitt [14]. Generally, our predictions for the critical temperature and other quantities are different from theirs since our couplings do not depend on

the chemical potentials. Secondly, we have not made a high-temperature approximation of the thermal integrals in the gap equations that determine the dressed masses. Finally, our gap equations and effective potential include renormalization effects.

One drawback of the 2PI Hartree approximation is that it does not obey Goldstone's theorem. We have shown this explicitly and quantified the deviation by the right-hand-side of Eq. (70). For practical purposes, the violation is negligible which is reassuring. We therefore believe that the 2PI Hartree approximation is a useful nonperturbative approximation for systems in thermal equilibrium.

The Hartree approximation and the large-N limit are both mean-field approximations. It would be desireable to go beyond mean field for example by including next-to-leading corrections in the $1/N$-expansion. In the context of pions, this has been done using the 1PI $1/N$-expansion, but only for vanishing chemical potential [51, 52]. Another method is the functional renormalization group method, see Refs. [53, 54, 55, 56] and references therein. The functional renormalization group method is a nonperturbative approach that has been very succesful in finite-temperature field theory. The essence of this approach is a flow equation for the average effective action. This flow equation cannot be solved exactly, but one must resort to approximations. A systematic approximation is the derivative expansion of the effective action and even the simplest truncation, namely the local-potential approximation, often yields good results. In the context of Bose condensation, functional renormalization group methods have been applied to the nonrelativistic case in Refs. [57, 58, 59, 60]. Work on pion and kaon condensation using these methods is in progress [61].

APPENDIX A: RENORMALIZATION

In this section, we first discuss the renormalization of the gap equations and effective potential for the $O(2N)$-symmetric models in detail. We then briefly skecth the renormalization of the gap equations of the $O(2)\times O(2)$-symmetric model.

1. $O(2N)$-symmetric models

The effective potential and the gap equations contain divergent sum-integrals. For the theory to be renormalizable, the divergent terms must be independent of the the temperature T and the chemical potential μ. Below we show explicitly that there are individual μ-dependent contributions to the gap equations which are divergent, but they cancel amongst themselves and so the counterterms are the same as those needed to renormalize the theory in the vacuum.

There is a complication regarding the renormalization of the quartic coupling in the 2PI Hartree approximation. The truncatation of the 2PI effective action allows one to define two independent four-point functions and thus two independent sets of counterterms associated with them [43]. They correspond to the two $O(N)$ invariants $\delta_{ij}\delta_{kl}$ and $\delta_{ik}\delta_{jl}+\delta_{ik}\delta_{jk}$ from which F_{ijkl} is built. More generally, each $O(N)$-invariant term in the 2PI effective action has its own independent counterterm. For example, the counterterms arising from the classical potential in Eq. (18) are written as

$$\frac{1}{2}\delta m_0^2+\frac{\delta\lambda_0}{8N}\phi_0^4\,, \tag{A1}$$

while the counterterms arising from the term $\mathrm{Tr}D_0^{-1}D$ are written as

$$\delta m_1^2 \;=\; \delta m^2+\frac{\delta\lambda_2^A+2\delta\lambda_2^B}{2N}\phi_0\,, \tag{A2}$$

$$\delta m_2^2 \;=\; \delta m^2+\frac{\delta\lambda_2^A}{2N}\phi_0\,. \tag{A3}$$

The two counterterms and $\delta\lambda^A$ and $\delta\lambda^B$. correspond to the two $O(N)$-invariant terms $\mathrm{Tr}[\phi_0^2]\mathrm{Tr}[D]$ and $\mathrm{Tr}[\phi_0^2 D]$ in the expression for $\mathrm{Tr}D_0^{-1}D$ that scale as N and one, respectively. It was shown in Ref. [42] that $\delta m_0^2=\delta m^2$ and $\delta\lambda_0=\delta\lambda^A+2\lambda^B$.

Renormalization of the gap equations can be done by applying an iterative procedure which is discussed in Refs. [41, 42]. To this end, we write the mass counterterm and the coupling constant counterterms δm^2, $\delta\lambda^A$, and $\delta\lambda^B$ as power series in λ:

$$\delta m^2 \;=\; \sum_{n=1}^{\infty}\delta m_n^2\,, \tag{A4}$$

$$\delta\lambda^A \;=\; \sum_{n=1}^{\infty}\delta\lambda_n^A\,, \tag{A5}$$

$$\delta\lambda^B \;=\; \sum_{n=1}^{\infty}\delta\lambda_n^B\,, \tag{A6}$$

where δm_n is a counterterm of order λ^n, and $\delta\lambda_n^A$ and $\delta\lambda_n^B$ are counterterms of order λ^{n+1}, respectively. Similarly, we write the self-energies Π_i as power series in λ:

$$\Pi_i \;=\; \Pi_i^{(1)}+\Pi_i^{(2)}+\cdots\,, \tag{A7}$$

where the superscript indicates the power of λ. Inserting Eqs. (A4)–(A7) into (25), we can determine $\Pi_n^{(n)}$, δm_n^2, $\delta\lambda_n^A$, and $\delta\lambda_n^B$ by iteration.

The Schwinger-Dyson equation (25) for the three diagonal components of the propagator is

$$D_{11}^{-1} = \left(D_0^{-1}\right)_{11} + \frac{\lambda}{2N}\left[3\sum\!\!\!\!\!\!\int_Q D_{11} + \sum\!\!\!\!\!\!\int_Q D_{22} + (2N-2)\sum\!\!\!\!\!\!\int_Q D_{33}\right] , \tag{A8}$$

$$D_{22}^{-1} = \left(D_0^{-1}\right)_{22} + \frac{\lambda}{2N}\left[\sum\!\!\!\!\!\!\int_Q D_{11} + 3\sum\!\!\!\!\!\!\int_Q D_{22} + (2N-2)\sum\!\!\!\!\!\!\int_Q D_{33}\right] , \tag{A9}$$

$$D_{33}^{-1} = \left(D_0^{-1}\right)_{33} + \frac{\lambda}{2N}\left[\sum\!\!\!\!\!\!\int_Q D_{11} + \sum\!\!\!\!\!\!\int_Q D_{22} + 2N\sum\!\!\!\!\!\!\int_Q D_{33}\right] . \tag{A10}$$

Since the off-diagonal parts of the self-energy vanish we have $(D_0)_{ij}^{-1} = (D)_{ij}^{-1}$ for $i \neq j$. The first iteration of the gap equation is found by ignoring the self-energy Π on the right-hand side of the gap equations (A8)–(A10), i.e. one replaces the full propagator by the tree-level propagator. If we denote the components of the free propagator by $(D_0)_{ij}$, we can write

$$M_1^2 - m_1^2 = \delta m^2 + \frac{\delta\lambda_1^A + 2\delta\lambda_1^B}{2N}\phi_0^2 + \frac{\lambda}{2N}\left[3\sum\!\!\!\!\!\!\int_Q (D_0)_{11} + \sum\!\!\!\!\!\!\int_Q (D_0)_{22} + (2N-2)\sum\!\!\!\!\!\!\int_Q (D_0)_{33}\right] , \tag{A11}$$

$$M_2^2 - m_2^2 = \delta m^2 + \frac{\delta\lambda_1^A}{2N}\phi_0^2 + \frac{\lambda}{2N}\left[\sum\!\!\!\!\!\!\int_Q (D_0)_{11} + 3\sum\!\!\!\!\!\!\int_Q (D_0)_{22} + (2N-2)\sum\!\!\!\!\!\!\int_Q (D_0)_{33}\right] , \tag{A12}$$

$$M_3^2 - m_3^2 = \delta m^2 + \frac{\delta\lambda_1^A}{2N}\phi_0^2 + \frac{\lambda}{2N}\left[\sum\!\!\!\!\!\!\int_Q (D_0)_{11} + \sum\!\!\!\!\!\!\int_Q (D_0)_{22} + 2N\sum\!\!\!\!\!\!\int_Q (D_0)_{33}\right] , \tag{A13}$$

where we have used Eqs. (A2)–(A3). The diagrammatic interpretation of the procedure is shown in Fig. 3. The propagator in the loops are free propagators and the first Feynman diagram on the right-hand side corresponds to the sum-integrals in Eqs. (A11)–(A13). We now consider in detail the renormalization of Eq. (A11). The sum-integrals can be split into divergent temperature-independent and temperature-dependent convergent parts and so we write

$$\sum\!\!\!\!\!\!\int_Q (D_0)_{nn} = I_n^d + I_n^T . \tag{A14}$$

The free propagator D_0 is obtained from Eq. (28) by replacing the medium-dependent masses M_i by m_i. The first sum-integral in Eq. (A11) then becomes

$$\sum\!\!\!\!\!\!\int_Q (D_0)_{11} = \frac{\omega_n^2 + p^2 + m_1^2 - \mu^2}{(\omega_n^2 + \omega_1^2)(\omega_n^2 + \omega_2^2)} . \tag{A15}$$

We begin by summing over the Matsubara frequences. This yields

$$\begin{aligned}\sum\!\!\!\!\!\!\int_Q (D_0)_{11} = \frac{1}{4}\int_q \Bigg[& \frac{1}{\omega_1}\left(1 + \frac{\frac{1}{2}(m_1^2 - m_2^2) + 2\mu^2}{\sqrt{4\mu^2\left[q^2 + \frac{1}{2}(m_1^2 + m_2^2)\right] + \frac{1}{4}(m_1^2 - m_2^2)^2}}\right)\coth\left(\frac{\omega_1}{2T}\right) \\ & + \frac{1}{\omega_2}\left(1 - \frac{\frac{1}{2}(m_1^2 - m_2^2) + 2\mu^2}{\sqrt{4\mu^2\left[q^2 + \frac{1}{2}(m_1^2 + m_2^2)\right] + \frac{1}{4}(m_1^2 - m_2^2)^2}}\right)\coth\left(\frac{\omega_2}{2T}\right)\Bigg] .\end{aligned} \tag{A16}$$

We are interested in isolating the divergent parts of the sum-integral. Dropping the convergent temperature-dependent parts of $\coth\left(\frac{\omega_{1,2}}{2T}\right)$ and rearranging, we obtain

$$I_1^d = \frac{1}{4}\int_q \left[\left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right) + \left(\frac{1}{\omega_1} - \frac{1}{\omega_2}\right)\frac{\frac{1}{2}(m_1^2 - m_2^2) + 2\mu^2}{\sqrt{4\mu^2\left[q^2 + \frac{1}{2}(m_1^2 + m_2^2)\right] + \frac{1}{4}(m_1^2 - m_2^2)^2}}\right] . \tag{A17}$$

For convenience, we introduce some shorthand notation:

$$E_1 \equiv \sqrt{q^2 + m_1^2} , \tag{A18}$$

$$A \equiv -\frac{1}{2}\left(m_1^2 - m_2^2\right) + \mu^2 \,, \tag{A19}$$

$$B \equiv \sqrt{4\mu^2\left[q^2 + \frac{1}{2}(m_1^2+m_2^2)\right] + \frac{1}{4}(m_1^2-m_2^2)^2} \,. \tag{A20}$$

The various factors in Eq. (A17) can then be compactly written as

$$\omega_{1,2}(q) = E_1\sqrt{1+\frac{A\pm B}{E_1^2}} \,, \tag{A21}$$

$$\frac{\frac{1}{2}(m_1^2-m_2^2)+2\mu^2}{\sqrt{4\mu^2\left[q^2+\frac{1}{2}(m_1^2+m_2^2)\right]+\frac{1}{4}(m_1^2-m_2^2)^2}} = \frac{-A+3\mu^2}{B} \,. \tag{A22}$$

The next step is to expand $\omega_{1,2}(q)$ in inverse powers of E. The integral I_d^1 can then be written as

$$\begin{aligned} I_1^d &= \frac{1}{4}\int_q \left\{ \frac{1}{E_1}\left[\left(1-\frac{A-B}{2E_1^2}+\frac{3(A-B)^2}{8E_1^4}+\cdots\right)+\left(1-\frac{A+B}{2E_1^2}+\frac{3(A+B)^2}{8E_1^4}+\cdots\right)\right]\right. \\ &\quad \left. -\frac{1}{E_1}\left[\left(1-\frac{A-B}{2E_1^2}+\frac{3(A-B)^2}{8E_1^4}+\cdots\right)-\left(1-\frac{A+B}{2E_1^2}+\frac{3(A+B)^2}{8E_1^4}+\cdots\right)\right]\frac{-A+3\mu^2}{B}\right\} \\ &= \frac{1}{4}\int_q \left[\frac{2}{E_1}-\frac{\mu^2(m_1^2-m_2^2)}{2E_1^5}+\cdots\right] \,. \end{aligned} \tag{A23}$$

We note that the integrals of the form $\int_q E_1^{-n}$ are divergent in the ultraviolet for $n \le 3$ and convergent for $n > 3$. Since the terms with $n = 3$ cancel in the integral, there is only one divergent term. We denote this term by I and it reads

$$I = \frac{1}{2}\int_q \frac{1}{\sqrt{q^2+m_1^2}} \,. \tag{A24}$$

This integral is easily calculated with dimensional regularization

$$I = -\frac{m_1^2}{(4\pi)^2}\left(\frac{\Lambda^2}{m_1^2}\right)^{\epsilon}\left[\frac{1}{\epsilon}+1+\mathcal{O}(\epsilon)\right] \,. \tag{A25}$$

Having isolated the UV divergence, the temperature-independenet part of the sum-integral in Eq. (A16) can now be written as

$$I_1^d = -\frac{m_1^2}{(4\pi)^2\epsilon} + I_1^c + \mathcal{O}(\epsilon) \,, \tag{A26}$$

where I_1^c is the finite part of I_1^d and is given by

$$I_1^c = \frac{1}{4}\int_q \left\{ \left[\frac{1}{\omega_1}+\frac{1}{\omega_2}\right] + \left[\frac{1}{\omega_1}-\frac{1}{\omega_2}\right]\frac{\frac{1}{2}(m_1^2-m_2^2)+2\mu^2}{\sqrt{4\mu^2\left[q^2+\frac{1}{2}(m_1^2+m_2^2)\right]+\frac{1}{4}(m_1^2-m_2^2)^2}} - \frac{2}{\sqrt{q^2+m_1^2}}\right\} - \frac{m_1^2}{(4\pi)^2}\left(L_1+1\right) \,, \tag{A27}$$

where $L_n = \log\frac{\Lambda^2}{m_n^2}$. The temperature-dependent part of the sum-integral (A16) is given by

$$\begin{aligned} I_1^T &= \frac{1}{2}\int_q \left\{ \left[\frac{1}{\omega_1(e^{\omega_1/T}-1)}+\frac{1}{\omega_2(e^{\omega_2/T}-1)}\right]\right. \\ &\quad \left. -\left[\frac{1}{\omega_1(e^{\omega_1/T}-1)}+\frac{1}{\omega_2(e^{\omega_2/T}-1)}\right]\frac{\frac{1}{2}(m_1^2-m_2^2)+2\mu^2}{\sqrt{4\mu^2\left[q^2+\frac{1}{2}(m_1^2+m_2^2)\right]+\frac{1}{4}(m_1^2-m_2^2)^2}}\right\} \,. \end{aligned} \tag{A28}$$

The integrals in Eqs. (A27)–(A28) are now finite in three dimensions and we set $\epsilon = 0$. The contributions to the right-hand side of Eq. (A11) from $(D_0)_{22}$ and $(D_0)_{33}$ are calculated in the same manner. The gap equation can then be written as

$$M_1^2 - m_1^2 = \delta m^2 + \frac{\delta\lambda_1^A\phi_0^2 + 2\delta\lambda_1^B}{2N}\phi_0^2 + \frac{\lambda}{2N}\Big\{ \left[3m_1^2 + m_2^2 + (2N-2)m_3^2\right]\frac{1}{\epsilon} + 3I_1^c + I_2^c + (2N-2)I_3^c. \\ +3I_1^T + I_2^T + (2N-2)I_3^T\Big\}. \tag{A29}$$

where I_2^c and I_2^T are obtained by exchanging m_1^2 for m_2^2 and vice versa in Eqs. (A27) and (A28), and

$$I_3^c = -\frac{m_3^2}{(4\pi)^2}\left(L_3 + 1\right), \tag{A30}$$

$$I_3^T = \frac{1}{2\pi^2}\int_0^\infty \frac{\mathrm{d}q q^2}{\sqrt{q^2+m_3^2}}\frac{1}{e^{\sqrt{q^2+m_3^2}/T}-1}\,. \tag{A31}$$

The divergences in Eq. (A29) are now cancelled by choosing the counterterms appropriately. This gives

$$\delta m_1^2 = \frac{\lambda}{(4\pi)^2\epsilon}\left(1+\frac{1}{N}\right)m^2\,, \tag{A32}$$

$$\delta\lambda_1^A = \frac{\lambda^2}{(4\pi)^2\epsilon}\left(1+\frac{2}{N}\right)\,, \tag{A33}$$

$$\delta\lambda_1^B = \frac{\lambda^2}{(4\pi)^2\epsilon}\frac{1}{N}\,. \tag{A34}$$

After renormalization, the first iteration of the gap equation for M_1 can be written as

$$M_1^2 = m_1^2 + \frac{\lambda}{2N}\left[3I_1^c + I_2^c + (2N-2)I_3^c \right. \\ \left. 3I_1^T + I_2^T + (2N-2)I_3^T\right]\,. \tag{A35}$$

The procedure is carried out iteratively to all orders in m_n^2 and λ. The final result is given in Eqs. (30)-(32) The counterterms δm_n^2 $\delta\lambda_n^A$, and $\delta\lambda_n^B$ are then expressed in terms of δm_{n-1}^2, $\delta\lambda_{n-1}^A$, and λ_{n-1}^B and obtain the recursion relations

$$\delta m_n^2 = \frac{\lambda}{(4\pi)^2\epsilon}\left(1+\frac{1}{N}\right)m_{n-1}^2\,, \tag{A36}$$

$$\delta\lambda_n^A = \frac{\lambda}{(4\pi)^2\epsilon}\left[\left(1+\frac{1}{N}\right)\delta\lambda_{n-1}^A + \frac{\delta\lambda_{n-1}^B}{N}\right] \tag{A37}$$

$$\delta\lambda_n^B = \frac{\lambda}{(4\pi)^2\epsilon}\frac{\delta\lambda_{n-1}^B}{N}\,. \tag{A38}$$

The counterterms are in agreement with those found by Fejos, Patkos, and Szep [42]. We note in particular that they are independent of temperature and chemical potential. They simplify significantly in the large-N limit, where $\delta\lambda^B = 0$. Renormalizing to all orders effectively means that we replace the tree-level masses m_i on the right-hand side of the gap equations by the medium dependent masses M_i. For each I_n in Eqs. (A27), (A28), and (A31) we therefore define a corresponding J_n, where the tree level masses m_n are replaced by the dressed masses M_n. The final result is given in Eqs. (30)–(32).

We next consider the gap equation (24). Differentiating Ω with respect to ϕ_0, we obtain unrenormalized gap equation

$$0 = \phi_0\left[m^2 - \mu^2 + \frac{\lambda}{2N}\phi_0^2 + \frac{\lambda}{2N}\left(3\sum\!\!\!\!\!\!\int_Q D_{11} + \sum\!\!\!\!\!\!\int_Q D_{22} + (2N-2)\sum\!\!\!\!\!\!\int_Q D_{33}\right)\right]\,. \tag{A39}$$

Renormalizing the gap equation (A39) in the same way as above, we obtain

$$0 = \phi_0\left[m^2 - \mu^2 + \frac{\lambda}{2N}\phi_0^2 + \frac{\lambda}{2N}\left[3(J_1^c + J_1^T) + (J_2^c + J_2^T) + (2N-2)(J_3^c + J_3^T)\right]\right]\,. \tag{A40}$$

We should make sure that the counterterms that renormalize the gap equations are also sufficient to renormalize the effective potential (18). We next consider separately the different terms contributing to Ω. The first term is $\mathrm{Tr}\log D^{-1}$. Taking the trace yields

$$\frac{1}{2}\mathrm{Tr}\log D^{-1} = \frac{1}{2}\sum\!\!\!\!\!\!\int_Q \log\left[\omega_n^2 + \tilde{\omega}_1^2(q)\right] + \frac{1}{2}\sum\!\!\!\!\!\!\int_Q \log\left[\omega_n^2 + \tilde{\omega}_2^2(q)\right] + (N-1)\sum\!\!\!\!\!\!\int_Q \log\left[\omega_n^2 + \tilde{\omega}_3^2(q)\right]\,. \tag{A41}$$

To carry out renormalization by the iterative procedure, we expand the sum-integrals in powers of the self-energies Π_i. To first order in the self-energies, we obtain

$$\begin{aligned}\frac{1}{2}\mathrm{Tr}\log D^{-1} &= \frac{1}{2}\sum\!\!\!\!\!\!\int_Q \log\left[\omega_n^2+\omega_1^2(q)\right]+\frac{1}{2}\sum\!\!\!\!\!\!\int_Q \log\left[\omega_n^2+\omega_2^2(q)\right]+(N-1)\sum\!\!\!\!\!\!\int_Q \log\left[\omega_n^2+\omega_3^2(q)\right]\\ &+\frac{1}{2}\Pi_1^{(1)}\sum\!\!\!\!\!\!\int_Q \frac{\omega_n^2+q^2+m_1^2+\mu^2}{(\omega_n^2+\omega_1^2)(\omega_n^2+\omega_2^2)}+\frac{1}{2}\Pi_2^{(1)}\sum\!\!\!\!\!\!\int_Q \frac{\omega_n^2+q^2+m_2^2+\mu^2}{(\omega_n^2+\omega_2^2)(\omega_n^2+\omega_2^2)}+(N-1)\Pi_3^{(1)}\sum\!\!\!\!\!\!\int_Q \frac{1}{\omega_n^2+\omega_3^2}\,. \quad \text{(A42)}\end{aligned}$$

Summing over the Matsubara frequencies, each term involving $\log(\omega_n^2+\omega_i^2(q))$ can be written in the form

$$\sum\!\!\!\!\!\!\int_Q \log\left[\omega_n^2+\omega_i^2(q)\right] = \mathcal{I}_n^d+\mathcal{I}_n^T\,, \quad \text{(A43)}$$

where

$$\mathcal{I}_n^d = \int_q \omega_i(q) \quad \text{(A44)}$$

$$\mathcal{I}_n^c = 2T\int_q \log\left[1-e^{-\omega_i(q)/T}\right]\,. \quad \text{(A45)}$$

We see right away that the temperature-dependent integrals are convergent. We now consider the temperature-independent term in Eq. (A44) where $n=1,2$. With the expected result in mind we rewrite $\omega_1+\omega_2=(\omega_1+\omega_2)/2+(\omega_1+\omega_2)/2$ and expand the terms in the first parantheses in $E_1=\sqrt{q^2+m_1^2}$ and the second in $E_2=\sqrt{q^2+m_2^2}$. This yields

$$\omega_1(q)+\omega_2(q)=E_1+E_2-\frac{1}{4}(m_1^2-m_2^2)\left(\frac{1}{E_1}-\frac{1}{E_2}\right)-\frac{1}{16}(m_1^2-m_2^2)^2\left(\frac{1}{E_1^3}+\frac{1}{E_2^3}\right)+\mathcal{O}(1/E_1^5)+\mathcal{O}(1/E_2^5) \quad \text{(A46)}$$

The first two terms are the divergences we get when $\mu=0$. As for the other terms, writing $E_2^2=E_1^2-(m_1^2-m_2^2)$ and expanding the resulting expression in E_1, we find that the divergent parts do in fact cancel:

$$-\frac{1}{4}(m_1^2-m_2^2)\left(\frac{1}{E_1}-\frac{1}{E_2}\right)-\frac{1}{16}(m_1^2-m_2^2)^2\left(\frac{1}{E_1^3}+\frac{1}{E_2^3}\right)+\mathcal{O}(1/E_1^5)+\mathcal{O}(1/E_2^5)=\mathcal{O}'(1/E_1^5)\,. \quad \text{(A47)}$$

This shows that the divergent term in each sum-integral is on the form

$$K_n = \int_q \sqrt{q^2+m_n^2}\,. \quad \text{(A48)}$$

With dimensional regularization, the integral can be easily calculated and reads

$$K_n = -\frac{m_n^4}{2(4\pi)^2}\left(\frac{\Lambda^2}{m_n^2}\right)^{\epsilon}\left[\frac{1}{\epsilon}+\frac{3}{2}\right]\,. \quad \text{(A49)}$$

To zeroth order in the self-energies Π_i, we can write

$$\frac{1}{2}\mathrm{Tr}\log D^{-1} = -\frac{1}{4(4\pi)^2}\left[m_1^4+m_2^4+2(N-1)m_3^2\right]\frac{1}{\epsilon}+\frac{1}{2}(\mathcal{I}_1^c+\mathcal{I}_1^T)+\frac{1}{2}(\mathcal{I}_2^c+\mathcal{I}_2^T)+(N-1)(\mathcal{I}_3^c+\mathcal{I}_3^T)\,, \quad \text{(A50)}$$

where

$$\mathcal{I}_{1,2}^c = \int_q [\omega_{1,2}(q)-E_{1,2}]-\frac{m_n^4}{2(4\pi)^2}\left[L_n+\frac{3}{2}\right]\,, \quad \text{(A51)}$$

$$\mathcal{I}_3^c = -\frac{m_n^4}{2(4\pi)^2}\left[L_3+\frac{3}{2}\right]\,. \quad \text{(A52)}$$

We next consider the term $\mathrm{Tr}D_0^{-1}D$. Using that $\Pi = D^{-1} - D_0^{-1}$, we obtain $\mathrm{Tr}D_0^{-1}D = \mathrm{Tr}I - \mathrm{Tr}\Pi D$, where I is the identity matrix. The first term vanishes in dimensional regularization and we are left with $\mathrm{Tr}\Pi D$. Since Π is diagonal, we obtain

$$\frac{1}{2}\mathrm{Tr}\Pi D = \frac{1}{2}\Pi_1 \sum\!\!\!\!\!\!\int_Q \frac{\omega_n^2+q^2+M_1^2+\mu^2}{(\omega_n^2+\tilde{\omega}_1^2)(\omega_n^2+\tilde{\omega}_2^2)} + \frac{1}{2}\Pi_2 \sum\!\!\!\!\!\!\int_Q \frac{\omega_n^2+q^2+M_2^2+\mu^2}{(\omega_n^2+\tilde{\omega}_2^2)(\omega_n^2+\tilde{\omega}_2^2)} + (N-1)\Pi_3 \sum\!\!\!\!\!\!\int_Q \frac{1}{\omega_n^2+\tilde{\omega}_3^2}\,. \quad \text{(A53)}$$

Since we are expanding the self-energies as $\Pi_i = \Pi_i^{(1)} + \Pi_i^{(2)} + ...$, the leading term is obtained by replacing Π_i by $\Pi_i^{(1)}$ and $\tilde{\omega}_i$ by ω_i. We then see that the leading term exactly cancels against the terms on the second line in Eq. (A42). At higher orders this no longer the case. The term $\mathrm{Tr}\Pi D$ then gives rise to an additional term which after renormalization reads

$$-\frac{1}{2}(M_1^2-m_1^2)(J_1^c+J_1^T) - \frac{1}{2}(M_2^2-m_2^2)(J_2^c+J_2^T) - (N-1)(M_3^2-m_3^2)(J_3^c+J_3^T)\,. \quad \text{(A54)}$$

Finally, we consider the two-loop diagrams in $\Phi[D]$. The terms are all given by products of one-loop sum-integrals that we have already calculated

$$\begin{aligned}\Phi[D] &= \frac{\lambda}{8N}\left\{\left[\sum\!\!\!\!\!\!\int_Q D_{11} + \sum\!\!\!\!\!\!\int_Q D_{22} + 2(N-1)\sum\!\!\!\!\!\!\int_Q D_{33}\right]^2\right. \\ &\left. +2\sum\!\!\!\!\!\!\int_Q D_{11}\sum\!\!\!\!\!\!\int_Q D_{11} + 2\sum\!\!\!\!\!\!\int_Q D_{22}\sum\!\!\!\!\!\!\int_Q D_{22} + 4(N-1)\sum\!\!\!\!\!\!\int_Q D_{33}\sum\!\!\!\!\!\!\int_Q D_{33}\right\}\,. \quad \text{(A55)}\end{aligned}$$

Expressing this in terms of the integrals I_i^d and I_i^T, we obtain

$$\begin{aligned}\Phi[D] &= \frac{\lambda}{8N}\left\{\left[\left(-\frac{m_1^2}{(4\pi)^2\epsilon}+I_1^c+I_1^T\right)+\left(-\frac{m_2^2}{(4\pi)^2\epsilon}+I_2^d+I_2^T\right)+2(N-1)\left(-\frac{m_3^2}{(4\pi)^2\epsilon}+I_3^d+I_3^T\right)\right]^2\right. \\ &\left. +2\left(-\frac{m_1^2}{(4\pi)^2\epsilon}+I_1^c+I_1^T\right)^2+2\left(-\frac{m_2^2}{(4\pi)^2\epsilon}+I_2^d+I_2^T\right)^2+4(N-1)\left(-\frac{m_3^2}{(4\pi)^2\epsilon}+I_3^d+I_3^T\right)^2\right\}\,. \quad \text{(A56)}\end{aligned}$$

The counterterm arising from $\frac{1}{2}\mathrm{Tr}D_0^{-1}D$ is denoted by $\frac{1}{2}\mathrm{Tr}\delta D_0^{-1}D$. Since the matrix δD_0^{-1} is diagonal, whose elements are given by δm_i^2, the matrix $\delta D_0^{-1}D$ is also diagonal. We then find

$$\frac{1}{2}\mathrm{Tr}\delta D_0^{-1}D = \frac{1}{2}\delta m_1^2\sum\!\!\!\!\!\!\int_Q D_{11} + \frac{1}{2}\delta m_2^2\sum\!\!\!\!\!\!\int_Q D_{22} + (N-1)\delta m_3^2\sum\!\!\!\!\!\!\int_Q D_{33}\,. \quad \text{(A57)}$$

Again the sum-integrals are known and the result is

$$\frac{1}{2}\mathrm{Tr}\delta D_0^{-1}D = \frac{1}{2}\delta m_1^2\left[\frac{m_1^2}{(4\pi)^2\epsilon}+I_1^T+I_1^c\right] + \frac{1}{2}\delta m_2^2\left[\frac{m_2^2}{(4\pi)^2\epsilon}+I_2^T+I_2^c\right] + (N-1)\delta m_3^2\left[\frac{m_3^2}{(4\pi)^2\epsilon}+I_3^T+I_3^c\right] \quad \text{(A58)}$$

Adding Eqs. (A50), (A56), and (A58) and using the counterterms in Eqs (A32)–(A34), all the divergences cancel and we obtain the renormalized effective potential to first order

$$\begin{aligned}\Omega^{(1)} &= \frac{1}{2}(m^2-\mu^2)\phi_0^2 + \frac{\lambda}{8N}\phi_0^4 + \frac{1}{2}(\mathcal{I}_1^c+\mathcal{I}_1^T) + \frac{1}{2}(\mathcal{I}_2^c+\mathcal{I}_2^T) + (N-1)(\mathcal{I}_3^c+\mathcal{I}_3^T) \\ &+\frac{3\lambda}{8N}(I_1^c+I_1^T)^2 + \frac{3\lambda}{8N}(I_2^c+I_2^T)^2 + \frac{\lambda}{2}(N-1)(I_3^c+I_3^T)^2 + \frac{\lambda}{4N}(I_1^c+I_1^T)(I_2^c+I_2^T) \\ &+\frac{\lambda}{2N}(N-1)(I_1^c+I_1^T)(I_3^c+I_3^T) + \frac{\lambda}{2N}(N-1)(I_2^c+I_2^T)(I_3^c+I_3^T)\,. \quad \text{(A59)}\end{aligned}$$

The full nonperturbative effective potential is now obtained by renormalizing to all orders in the coupling constant. The final result is then given by Eq. (37), where the integrals $\mathcal{J}_i^c$ and $\mathcal{J}_i^T$ are obtained from $\mathcal{I}_i^c$ and $\mathcal{I}_i^T$ by the replacements $m_i \to M_i$.

2. $O(2) \times O(2)$-symmetric models

The gap equations for the dressed masses and the background field ϕ_0 follow from Eqs. (25) and (64):

$$M_1^2 - m_1^2 = \delta m_0^2 + \frac{\delta\lambda_0^A + 2\delta\lambda_0^B}{2}\phi_0^2 + \frac{1}{2}\sum\!\!\!\!\!\!\int_Q \left[3\lambda_0 D_{11} + \lambda_0 D_{22} + \lambda_H D_{33} + \lambda_H D_{44}\right] , \tag{A60}$$

$$M_2^2 - m_2^2 = \delta m_0^2 + \frac{\delta\lambda_0^A}{2}\phi_0^2 + \frac{1}{2}\sum\!\!\!\!\!\!\int_Q \left[\lambda_0 D_{11} + 3\lambda_0 D_{22} + \lambda^H D_{33} + \lambda^H D_{44}\right] , \tag{A61}$$

$$M_3^2 - m_3^2 = \delta m_+^2 + \frac{\delta\lambda_H}{2}\phi_0^2 + \frac{1}{2}\sum\!\!\!\!\!\!\int_Q \left[\lambda_H D_{11} + \lambda_H D_{22} + 3\lambda_+ D_{33} + \lambda_+ D_{44}\right] , \tag{A62}$$

$$M_4^2 - m_3^2 = \delta m_+^2 + \frac{\delta\lambda_H}{2}\phi_0^2 + \frac{1}{2}\sum\!\!\!\!\!\!\int_Q \left[\lambda_H D_{11} + \lambda_H D_{22} + \lambda_+ D_{33} + 3\lambda_+ D_{44}\right] , \tag{A63}$$

$$0 = \phi_0 \left[\mu^2 - m^2 + 3\lambda_0 \sum\!\!\!\!\!\!\int_Q D_{11} + \lambda_0 \sum\!\!\!\!\!\!\int_Q D_{22} + \lambda_H \sum\!\!\!\!\!\!\int_Q D_{33} + \lambda_H \sum\!\!\!\!\!\!\int_Q D_{44}\right] . \tag{A64}$$

Note that we have two counterterms λ_0^A and λ_0^B for λ_0 that correspond to the two $O(2)$-invariant terms in the effective action.

Again renormalization can be carried out by the iterative procedure of Refs. [41, 42]. The first step consists of neglecting the self-energies in the gap equations. The counterterms necessary to render them finite are given by

$$\delta m_{0(1)}^2 = \frac{1}{(4\pi)^2\epsilon}\left[2\lambda_0 m_0^2 + \lambda_H m_+^2\right] , \tag{A65}$$

$$\delta m_{+(1)}^2 = \frac{1}{(4\pi)^2\epsilon}\left[2\lambda_+ m_+^2 + \lambda_H m_0^2\right] , \tag{A66}$$

$$\delta\lambda_{0(1)}^A = \frac{1}{(4\pi)^2\epsilon}\left(3\lambda_0^2 + \lambda_H^2\right) , \tag{A67}$$

$$\delta\lambda_{0(1)}^B = \frac{\lambda_0^2}{(4\pi)^2\epsilon} , \tag{A68}$$

$$\delta\lambda_{H(1)} = \frac{2}{(4\pi)^2\epsilon}\left(\lambda_H\lambda_0 + \lambda_H\lambda_+\right) , \tag{A69}$$

where the index (1) indicates that this is the first iteration. The iterative procedure leads to the following recursion relations for the counterterms

$$\delta m_{0(n)}^2 = \frac{1}{(4\pi)^2\epsilon}\left[\left(\delta\lambda_{0(n-1)}^A + \delta\lambda_{0(n-1)}^B\right) m_0^2 + \delta\lambda_{H(n-1)} m_+^2\right] , \tag{A70}$$

$$\delta m_{+(n)}^2 = \frac{1}{(4\pi)^2\epsilon}\left[2\delta\lambda_{+(n-1)} m_+^2 + \delta\lambda_{H(n-1)} m_0^2\right] , \tag{A71}$$

$$\delta\lambda_{0(n)}^A = \frac{1}{(4\pi)^2\epsilon}\left[\lambda_0\left(2\delta\lambda_{0(n-1)}^A + \delta\lambda_{0(n-1)}^B\right) + \lambda_H\delta\lambda_{H(n-1)}\right] , \tag{A72}$$

$$\delta\lambda_{0(n)}^B = \frac{1}{(4\pi)^2\epsilon}\lambda_0\delta\lambda_{0(n-1)}^B , \tag{A73}$$

$$\delta\lambda_{+(n)} = \frac{1}{(4\pi)^2\epsilon}\left[2\lambda_+\delta\lambda_{+(n-1)} + \frac{1}{2}\lambda_H\delta\lambda_{H(n-1)}\right] , \tag{A74}$$

$$\delta\lambda_{H(n)} = \frac{1}{(4\pi)^2\epsilon}\left[\lambda_H\left(\delta\lambda_{0(n-1)}^A + \delta\lambda_{0(n-1)}^B\right) + \lambda_H\left(\delta\lambda_{+(n-1)}^A + \delta\lambda_{+(n-1)}^B\right)\right] . \tag{A75}$$

The dressed masses M_3 and M_4 are equal. This follows from subtracting Eq (A63) from Eq (A62) and renormalizing the resulting expression order by order and using the fact that the tree-level masses m_3 and m_4 are equal.

Clearly, this holds only as long as there is no charged condensate.

These recursion relations are in agreement with those found by Fejos, Patkos, and Szep [42] for the $O(N) \times O(M)$-symmetry after having set $N = M = 2$. The final result for the gap equations is given in Eqs. (65)–(68). Finally, the counterterms needed to render the gap equations finite are also those needed to renormalize the effective potential. The final result for the effective potential is given by Eq. (71).

ACKNOWLEDGMENT

J.O.A would like to thank M. Alford and T. Brauner for useful discussions.

[1] M. G. Alford, K. Rajagopal, and F. Wilczek, Nucl. Phys. **B537**, 443 (1999).

[2] F. Wilczek, Lectures given at 9th CRM Summer School: Theoretical Physics at the End of the 20th Century, Banff, Alberta, Canada, 27 Jun - 10 Jul 1999. Published in *Banff 1999, Theoretical physics at the end of the twentieth century* 567. e-Print: hep-ph/0003183.

[3] K.Rajagopal and F. Wilczek, Chapter 35 in the Festschrift in honor of B.L. Ioffe, "At the Frontier of Particle Physics / Handbook of QCD", M. Shifman, ed., (World Scientific). In *Shifman, M. (ed.): At the frontier of particle physics, vol. 3* 2061-2151. e-Print: hep-ph/0011333.

[4] D. H. Rischke, Prog. Part. Nucl. Phys. **52**, 197 (2004).

[5] M. A. Stephanov, PoS LAT2006, 024 (2006).

[6] L. McLerran and R. D. Pisarski, Nucl. Phys. **A796**, 83 (2007).

[7] M. G. Alford, A. Schmitt, and K. Rajagopal, arXiv:0709.4635. To appear in Rev. Mod. Phys.

[8] P. Braun-Munzinger and J. Wambach, arXiv:0801.4256.

[9] R. Casalbuoni and R. Gatto, Phys. Lett. **B464** 111.

[10] D. T. Son and M. A. Stephanov, Phys. Rev. D **61**, 074012 (2001); ibid D **62**, 059902 (2002).

[11] P. F. Bedaque and T. Schäfer, Nucl. Phys. A **697**, 802 (2002).

[12] D. B. Kaplan and S. Reddy, Phys. Rev. D **65**, 054042(2002).

[13] T. Schäfer , Phys. Rev. D **65**, 094033 (2002).

[14] M. G. Alford, M. Braby, and A. Schmitt, J. Phys. **G**: nucl. Part. Phys. **35**, 025002 (2008).

[15] M. Buballa, Phys. Lett. **B609**,57 (2005).

[16] M. M. Forbes, Phys. Rev. **D72**, 094032 (2005).

[17] D. Ebert and K. G. Klimenko, Phys. Rev. D **75**, 045005 (2007).

[18] D. Ebert, , K. G. Klimenko, and V. L. Yudichev, Eur. Phys. J. **C53**, 65 (2008).

[19] M. Ruggieri, JHEP **0707**, 031 (2007).

[20] D. Ebert and K.G. Klimenko, arXiv:0809.5254.

[21] V. Kleinhaus, M. Buballa, D. Nickel, and M. Oertel, Phys. Rev. D **76**, 074024 (2007).

[22] H. J. Warringa, hep-ph/0606063.

[23] T. Schäfer, D. T. Son, M. A. Stephanov, D. Toublan, and J. J. M. Verbaarschot, Phys. Lett. **B522**, 67 (2001).

[24] V. A. Miransky and I. A. Shovkovy, Phys. Rev. Lett. **88**, 111601 (2002).

[25] V. P. Gusynin, V. A. Miransky, and I. A. Shovkovy, Mod. Phys. Lett. **A19**, 1341 (2004).

[26] M. Alford and K. Rajagopal, JHEP, **0206**, 031 (2002).

[27] I. A. Shovkovy, Found. Phys. **35**, 1309 (2005).

[28] D. D. Dietrich and D. H. Rischke, Prog. Part. Nucl. Phys. **53**, 305 (2004).

[29] M. Buballa and I. A. Shovkovy, Phys. Rev. D **72**, 097501 (2005).

[30] D. Blaschke, S. Fredriksson, H. Grigorian, A.M. Oztas, and F. Sandin, Phys. Rev. D **72**, 065020 (2005).

[31] J. M. Cornwall, R. Jackiw, and E. Tomboulis, Phys. Rev. **D10**, 2428 (1974).

[32] G. Amelino-Camelia, Phys. Lett. **B407**, 268 (1997).

[33] N. Petropoulos, J. Phys.**G25**, 2225 (1999).

[34] J. T. Lenaghan and D. H. Rischke, J. Phys. **G26**, 431 (2000).

[35] E. Braaten and E. Petitgirard, Phys. Rev. D **65**, 041701(R) (2002); Phys.Rev. D **65**, 085039 (2002).

[36] J. O. Andersen and M. Strickland, Phys. Rev. D **71**, 025011 (2005).

[37] J. Berges, Sz. Borsanyi, U. Reinosa, and J. Serreau, Phys. Rev. D **71**, 105004 (2005).

[38] S. Borsanyi and U. Reinosa, Phys. Lett. **B661**, 88 (2008).

[39] J. O. Andersen, Phys. Rev. D **75**, 065011 (2007).

[40] H. van Hees, J. Knoll, Phys. Rev. D **65**, 025010 (2002); ibid **65**, 105005 (2002); **65**, 025028 (2002).

[41] J.-P. Blaizot, E. Iancu and U. Reinosa, Phys. Lett. B **568**, 160 (2003), Nucl. Phys. A **736**, 149 (2004).

[42] G. Fejos, A. Patkos, and Zs. Szep, Nucl. Phys. A **803**, 115 (2008).

[43] J. Berges, Sz. Borzányi, U. Reinosa, and J. Serreau, Ann. Phys. **320**, 344 (2005).

[44] H. E. Haber and H. A. Weldon, Phys. Rev. D **25**, 502, (1982).

[45] G. Aarts, D. Ahrensmeier, R. Baier, J. Berges, and J. Serreau, Phys. Rev. D **66**, 045008, (2002).

[46] D. T. Son and M. Stephanov, Phys. Rev. Lett. **86**, 592 (2001).

[47] M. Alford, J. Berges, and J. M. Cheyne, Phys. Rev. D **70**, 125002 (2004).

[48] H. B. Nielsen and S. Chadha, Nucl. Phys. **B105**, 445 (1976).

[49] T. Brauner, Phys. Rev. D **72**, 076002 (2005); PhD Thesis, Nuclear Physics Institute, Prague (2006), hep-ph/0606300.

[50] H. Abuki, R. Anglani, R. Gatto, M. Pellicoro, and M. Ruggieri, arXiv:0809.2658.

[51] J. O. Andersen, D. Boer, and H. J. Warringa, Phys. Rev.

D **69**, 076006 (2004); ibid **70**, 116007 (2004).
[52] J. O Andersen and T. Brauner, Phys. Rev. D **66**, 045008, (2008).
[53] C. Wetterich, Phys. Lett. **B301**, 90 (1993).
[54] J. Berges, N. Tetradis, and C. Wetterich, Phys. Rept. **363**, 223 (2002).
[55] H. Gies, Presented at ECT* School on Renormalization Group and Effective Field Theory Approaches to Many-Body Systems, Trento, Italy, 27 Feb - 10 Mar 2006, hep-ph/0611146
[56] B. Delamotte, cond-mat/0702365.
[57] J. O. Andersen and M. Strickland, Phys. Rev. A **60**, 1442 (1999).
[58] J. O. Andersen, Rev. Mod. Phys. **76**, 599 (2004).
[59] S. Floerchinger and C. Wetterich, Phys. Rev. A **77**, 053603 (2008).
[60] J.-P. Blaizot, R. M. Galain, and N. Wschebor Europhys. Lett. **72**, 705 (2005); J.-P. Blaizot, Lectures given at ECT* School on Renormalization Group and Effective Field Theory Approaches to Many-Body Systems, Trento, Italy, 27 Feb-10 Mar 2006.
[61] J. O. Andersen and L. E. Leganger, in progress.

This figure "p.gif" is available in "gif" format from:

http://arXiv.org/ps/0810.5510v1